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MEASUREMENTS OF THE VARIATION WITH DEPTH OF THE MAIN GEOMAGNETIC FIELD

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The main geomagnetic field is attributable either to some deep-seated phenomena within the earth or to a fundamental property of rotating matter in which the source of the field would be distributed throughout the whole earth. The two types of explanation predict a different variation of the components of the main field with depth within the crust and can be tested by direct measurement in suitable localities. Measurements in five mines in northern England are presented and discussed, and they provide evidence in favour of the core theory.

List of symbols used

F denotes the magnetic field vector.

H denotes the horizontal component of the geomagnetic field.

Z denotes the vertical component of the geomagnetic field, positive downwards.

X denotes the northward horizontal component of the geomagnetic field, positive northwards.

Y denotes the eastward horizontal component of the geomagnetic field, positive eastwards.

D denotes the angle of declination.

x and y denote distances on the earth's surface in the northward and eastward directions respectively.

z denotes a distance from a point on the surface in a direction vertically downwards.

R denotes the radius of the earth.

Points are denoted by the polar co-ordinates based on the geocentre; r the radial distance, θ the angle of colatitude and λ the angle of longitude, positive eastwards.

The values of field intensities given in the tables and figures are in units of 1γ ($= 10^{-5}$ gauss).

National grid references are used throughout the paper.

1. Introduction

In the absence of a tenable explanation of the main geomagnetic field, the discovery of the axial magnetic fields of the sun and of the star 78 Virginis, by Hale (1913) and H. W. Babcock (1947*a*) respectively, suggested that these three fields might be examples of a fundamental property of masses in rotation. Schuster (1912) and Wilson (1923) had shown that the magnetic moments (P) and the angular momenta (U) of the sun and the earth were approximately in the same ratio, and Blackett (1947) and Babcock (1947*b*) showed that the relation

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was true also for 78 Virginis. Blackett's demonstration that P/U was nearly equal to a simple ratio involving only the gravitational constant (G) and the velocity of light (c) seemed to support the view that the phenomena were not the result of internal processes depending on the ordinary electrical or magnetic properties of the material. Blackett therefore drew the tentative conclusion that the relation

$$P = \beta G^{\frac{1}{2}} U/2c \quad (\beta \doteqdot 1) \tag{1}$$

represented a new property of matter, the nature of which might only be fully understood when a unified theory of the electromagnetic and gravitational fields was developed. H. W. Babcock (1950) has now shown that more than twenty-five of the peculiar stars and spectrum variables of spectral types A and early F have surface magnetic fields of the order of a few thousand gauss. That the fields of some of the spectrum variables fluctuate in intensity has been held to disprove equation (1), but Runcorn (1948*a*, 1950) and Blackett (1949) have pointed out that the most likely explanation of this is an interaction between the mechanical oscillations in the star and its steady field, the latter approximately satisfying equation (1). Such a mechanism has been examined in detail by Schwarzschild (1949) and Gjellestad (1950). A more serious setback to a fundamental theory has been the recent failure to find a solar magnetic field as large as Hale's original work suggested. H. D. Babcock (1948) reported the polar field to be less than 5 gauss on twenty-four occasions between 1939 and 1945 and between 5 and 60 gauss on eighteen occasions, while Thiessen (1949) and Von Kluber (1951) did not find a field in 1949 greater than about 1 gauss, not significantly outside the experimental error of their apparatus. The explanation of the intense magnetic fields of sun-spots and the characteristic form of the solar corona seem to require the existence of a general solar field, but it now seems doubtful whether its magnitude is consistent with equation (1).

In 1947 Dr E. C. Bullard suggested that such a fundamental hypothesis, or distributed theory as we shall call it, might be tested directly for the earth, as it might be expected to predict a variation of the field within the earth's crust different from the inverse cube law predicted by other theories. These latter, which will be termed 'core theories', suppose that there are no sources of the field in the upper part of the mantle, except for the anomalies produced by magnetized rocks. The most important of the core theories postulate electric current flow in the *core* of the earth, which is thought to be metallic in character with an electrical conductivity high compared with that of the mantle; an example of a specific mechanism by which the currents might be maintained is the self-excited dynamo theory suggested by Elsasser (1946*a*, *b*, 1947) and Bullard (1948). Runcorn (1948*b*) and Chapman (1948*a*) calculated the variation of the field intensities with depth throughout the earth's interior for the fundamental theory, but for the purpose of the discussion here we will derive formulae applicable to small depths only. No acceptable theoretical basis of equation (1) has yet been suggested, so that its consequences have been worked out by analogy with classical magnetostatics. Suppose it is assumed that equation (1) holds for all massive rotating bodies. Then it follows that each mass element makes a contribution to the magnetic moment, as well as to the angular momentum, depending on its position in the body. Blackett (1947) suggested as a possible interpretation of equation (1) that each such element gives rise to virtual currents which have a magnetic effect similar to electric currents but which are not associated with the movement of real charges. Such virtual currents would produce a magnetic field ($\mathbf{F}$) according to the equation

$$\operatorname{curl}\mathbf{F} = 4\pi\mathbf{J}, \tag{2}$$

where $\mathbf{J}$ is the virtual current density vector, which considerations of symmetry show to be directed along lines of latitude. The contribution dP made to the total dipole by a spherical shell of matter at radius a and of thickness da, found by differentiating equation (1), is given by

$$dP = 4\pi\beta G^{\frac{1}{2}}\rho a^4\omega da/3c,$$

when ρ is the density at radius a and ω the angular velocity of the earth. Now the current density distribution ($\mathbf{i}$) on a spherical surface of radius a which gives externally an axial dipole field of moment M is given by the following expression (see, for instance, Chapman & Bartels 1940)

$$4\pi i_\lambda = 3M\sin\theta/a^3.$$

Therefore the virtual current density is given by

$$J = \beta G^{\frac{1}{2}}\rho a\omega\sin\theta/c. \tag{3}$$

For a distributed theory the values of H and Z at the surface and at a depth d will be the sum of the separate contributions of the surface shell of thickness d and of the inner sphere of radius $(R-d)$. Let the contributions made by the outer shell to the vertical and horizontal intensities just outside and just inside the shell be z_0, h_0, z_d and h_d respectively. Let the total vertical and horizontal components at the surface be Z_0 and H_0 and at depth d be Z_d and H_d. Then the contribution to Z_0 made by the inner sphere of radius $(R-d)$ at the surface will be Z_0-z_0 and the contribution at depth d will be Z_d-z_d. Hence

$$Z_d-z_d = (Z_0-z_0)\,(1+3d/R) \quad \text{if} \quad d \ll R.$$

Because the field is solenoidal, $z_0 = z_d$ if $d \ll R$. Therefore

$$Z_d = Z_0(1+3d/R). \tag{4}$$

Similarly $$H_d-h_d = (H_0-h_0)\,(1+3d/R).$$

By applying equation (2) we obtain

$$(h_0-h_d) = 4\pi J_0 d,$$

where J_0 is the virtual current density at the surface. Therefore

$$H_d = H_0(1+3d/R)-4\pi J_0 d.$$

Using equation (3) we obtain

$$H_d = H_0(1+3d/R-15\rho_s d/k\rho_m R), \tag{5}$$

where ρ_s is the density of surface rocks and $k\rho_m$ is a weighted mean density of the earth given by

$$k\rho_m = \int_0^R \rho a^4 da \Big/ \int_0^R a^4 da.$$

Now $\rho_s = 2{\cdot}7$ and $k\rho_m = 4{\cdot}8$ approximately, and therefore

$$H_d = H_0(1 - 6{\cdot}5d/R). \tag{6}$$

For a core theory the corresponding relations are

$$Z_d = Z_0(1 + 3d/R), \tag{7}$$

$$H_d = H_0(1 + 3d/R). \tag{8}$$

$Z_d - Z_0$ and $H_d - H_0$ will be denoted by ΔZ and ΔH respectively. Table 1 shows the values of ΔZ and ΔH calculated for England.

TABLE 1

depth (ft.)	core theory		distributed theory	
	ΔZ	ΔH	ΔZ	ΔH
2000	+13	+6	+13	−12
4000	+25	+11	+25	−23

The disturbances in the field intensities which arise from the magnetism of surrounding rocks is never less than 1 γ, which is also the sensitivity of easily available instruments, so that the direct measurement of the variation of the field with depth requires a mine or borehole at least 2000 ft. deep. The deepest coalmine in England is 4000 ft. deep, while the deepest mine in the world is in Mysore, India, which has workings 9000 ft. deep. The deepest boreholes in the world reach depths of 20,000 ft., but very few would be suitable for magnetic measurements because they are usually lined with steel casing. Measurements might be made in the sea to great depths, but the magnetic field of the induced currents due to the fluid motions might mask the effect. Since 1948 experiments have been made in deep mines in this country to attempt to distinguish between the two theories, and this paper presents the results of this work. Simultaneous measurements of H and Z were made between underground base stations in the mines and surface base stations vertically above these. A summary of the earlier results reaching conclusions similar to those of this paper has been published (Runcorn, Benson, Moore & Griffiths 1950).

The measurements of ΔH and ΔZ have to be corrected for the effects of local and man-made anomalies before they can be compared with the predicted values. Even so, as the field over the surface of the earth is not exactly that of an axial dipole at the geocentre, which is the assumption of the calculations above, the experimental results are relevant only if it can be shown that the gradients associated with the non-dipole field are either small enough not to affect the results or such that an allowance may be made for them.

2. THE ESTIMATION OF LOCAL ANOMALIES

The residual field at the surface of the earth obtained by the subtraction of that axial dipole which best fits the observed field shows anomalies of continental extent, known as *regional* anomalies, the average space gradients of which are about 1 γ/km. By their extent and by their close connexion with the geomagnetic secular change, it seems likely that they arise from some process in the core, such as electromagnetic induction. The field gradients measured over a few kilometres on the continents are often much larger, and in some parts of this country are of the order of 100 γ/km. (Falcon & Tarrant 1951). The extent of these *local* anomalies is of the order of tens of kilometres, and they are therefore likely to result from the permanent and induced magnetism of that outer part of the crust, about 30 km. thick, the temperature of which is below the Curie point. Anomalies of intermediate extent appear to be absent (Deel 1945). Over much of the continental sectors of the globe sedimentary rocks have been deposited in nearly horizontal strata to an average thickness of 3 km. The amount of magnetite they contain is very variable, but on the average it is of the order of 0·01 % and consequently their polarization is often negligible. These sediments are underlain by a shell of an average thickness of 15 km. which is in the main granitic but which is intruded from below in places by basic material. In the granites the magnetite content usually lies between 0·5 and 2 %, and in basic igneous rocks it may be as much as 10 %. In many places the origin of the local anomalies is thought to lie in the high relief and the variable magnetic polarization of the igneous basement (cf. Peters 1949). Assuming the sediments to be non-magnetic, the anomalous gradients at the surface of the earth will be linear over distances small compared with the depth of the basement. In places where this simple scheme is applicable, the effect of anomalies on the radial variation of the main field can be examined and, if necessary, eliminated.

The method depends on the fact that, in the absence of current flow and of magnetic matter at a point, certain relations exist between the horizontal and vertical gradients of the field components, because the following equations are satisfied:

$$\operatorname{curl}\mathbf{F} = 0, \quad \operatorname{div}\mathbf{F} = 0.$$

It is necessary to use spherical co-ordinates and not Cartesian co-ordinates in this discussion, since the accuracy to which the Z and H components are measured in the experiments is of the order of the angle subtended at the geocentre by a distance on the surface of 1 km. At a point (r, θ, λ) the expressions for curl **F** and div **F** are as follows:

$$\operatorname{curl}\mathbf{F} = \frac{1}{r\sin\theta}\left[\frac{d}{d\theta}(\sin\theta\, Y)+\frac{dX}{d\lambda}\right]\mathbf{i}+\frac{1}{r}\left[-\frac{1}{\sin\theta}\frac{dZ}{d\lambda}-\frac{d}{dr}(rY)\right]\mathbf{j}+\frac{1}{r}\left[-\frac{d}{dr}(rX)+\frac{dZ}{d\theta}\right]\mathbf{k}, \tag{9}$$

and

$$\operatorname{div}\mathbf{F} = -\frac{1}{r^2}\frac{d}{dr}(r^2 Z)-\frac{1}{r\sin\theta}\frac{d}{d\theta}(\sin\theta\, X)+\frac{1}{r\sin\theta}\frac{dY}{d\lambda}, \tag{10}$$

where **i**, **j** and **k** are the unit vectors in the directions $(-z)$, $(-x)$ and y, where the negative direction of x and z is taken so that a right-handed system of co-ordinates is obtained. From equations (9) and (10) we obtain

$$\frac{dX}{dy} = \frac{dY}{dx} - \cot\theta\frac{Y}{R},$$

$$\frac{dY}{dz} = \frac{dZ}{dy} + \frac{Y}{R},$$

$$\frac{dX}{dz} = \frac{dZ}{dx} + \frac{X}{R},$$

$$\frac{dZ}{dz}+\frac{dX}{dx}+\frac{dY}{dy} = \cot\theta\frac{X}{R}+\frac{2Z}{R} = \frac{5Z}{2R}.$$

It is convenient to subtract from each of these gradients those due to the axial dipole component of the earth's field. These gradients are

$$\frac{dZ}{dx} = +\frac{2X}{R} \quad \frac{dZ}{dz} = +\frac{3Z}{R} \quad \frac{dZ}{dy} = 0,$$

$$\frac{dX}{dx} = -\frac{Z}{2R} \quad \frac{dX}{dz} = +\frac{3X}{R} \quad \frac{dX}{dy} = 0,$$

$$\frac{dY}{dx} = 0 \qquad \frac{dY}{dz} = 0 \qquad \frac{dY}{dy} = 0.$$

Then if the suffix a denotes the gradient due to an anomaly the following relations are obtained:

$$\left(\frac{dY}{dz}\right)_a = \frac{Y}{R} + \left(\frac{dZ}{dy}\right)_a, \tag{11}$$

$$\left(\frac{dX}{dz}\right)_a = \left(\frac{dZ}{dx}\right)_a, \tag{12}$$

$$\left(\frac{dX}{dy}\right)_a = \left(\frac{dY}{dx}\right)_a - \cot\theta\,\frac{Y}{R}, \tag{13}$$

$$\left(\frac{dZ}{dz}\right)_a + \left(\frac{dY}{dy}\right)_a + \left(\frac{dX}{dx}\right)_a = 0. \tag{14}$$

In practice it is more convenient to measure H than X and Y, but as Y is considerably less than X we can write, using equation (12),

$$\begin{aligned}\left(\frac{dH}{dz}\right)_a &= \frac{X}{H}\left(\frac{dX}{dz}\right)_a + \frac{Y}{H}\left(\frac{dY}{dz}\right)_a \\ &= \left(\frac{dZ}{dx}\right)_a.\end{aligned} \tag{15}$$

Assuming $dY/dy < dX/dx$ we obtain from equation (14)

$$\left(\frac{dZ}{dz}\right)_a = -\left(\frac{dX}{dx}\right)_a = -\left(\frac{dH}{dx}\right)_a. \tag{16}$$

The experimentally determined values of ΔH and ΔZ can thus be corrected, by data from surface surveys, for the effect of anomalous gradients, provided that these gradients are linear over the depth of the mine. If the field gradients determined by a surface survey over distances comparable to the depth are linear, and if the depth from which the anomalies arise is sufficiently great, this assumption will be justified. If the measured gradient agrees closely with the gradient which would arise from an axial dipole, the area may be said to be free from anomaly, and measurements in such areas are most likely to be useful, for where the anomalous gradients are large they are likely also to be non-linear.

If highly magnetized rocks lie too shallow it may happen that the surface gradients of the field components are apparently linear but that the vertical gradients between the surface and underground sites are substantially non-linear. Then the corrections for the anomalous surface gradients discussed above would not suffice to remove anomalies from the values of ΔZ and ΔH measured. An estimate is therefore required of the depth, relative to the depth a

of the underground site, above which extensive igneous rocks must not lie, if the above methods are to be applied. There seem to be two cases to consider. First, where the surface of the basement is plane and the polarization vector, assumed to be solenoidal within the basement, varies laterally, and secondly, where the polarization is constant and the surface of the basement is undulating. Both cases are similar mathematically and the latter will be considered.

If at a horizontal distance x the depth of the basement is $(b+a\cos 2\pi x/t)$, then the vertical component of the field at the surface will be given by

$$Z = \frac{2\pi ap}{t}\,\mathrm{e}^{-2\pi b/t}\cos 2\pi x/t,$$

where p is the polarization. If t is of the order of b, the field at the surface will be linear over distances comparable with t. The non-linear difference in the field intensities between the underground and surface sites will be given by

$$\Delta Z' = \frac{4\pi^3 apd^2}{t^3}\,\mathrm{e}^{-2\pi b/t}.$$

This will be greatest when $t = 2\pi b/3$ and the condition that it is less than 1γ, assuming a value for p of 10^{-3}, only rarely exceeded, is

$$b^3 > 148ad^2.$$

Thus if $a = 1000$ ft., $d = 2500$ ft., then b must exceed 9700 ft.

It is thus necessary to show that extensive bodies of igneous rocks are not likely to lie at shallower depths than this, if the method of correcting for anomalous gradients set out in this section is to be valid.

3. Geological aspects of the experiment

3·1. *The thickness of the sedimentary cover*

For reasons which will be discussed later in the paper the areas in which successful experiments were carried out were the western part of the South-east Lancashire Coalfield and the South Yorkshire Coalfield. It is necessary to make an estimate, based entirely on geological evidence, of the probable minimum depth at which extensive igneous rocks of the basement are likely to occur.

In these areas there is no evidence from mining operations or from borings as to the nature of the strata below the Middle Coal Measures. It is necessary to infer the probable succession and the thicknesses of the beds from a knowledge of the general stratigraphy and structure of the region. This was obtained from the relevant memoirs and regional handbooks of the Geological Survey, and from sources to which acknowledgement is made in the text. Figure 2*a* shows the geological structure of the regions concerned, together with the localities of boreholes from which information about the subsurface was obtained and the positions of the collieries where the measurements were made. Figure 2*b* gives the strata sections from boreholes and colliery shafts with some general information appropriate to these areas. The figures given for thicknesses must be considered as rather approximate.

Western part of the South-east Lancashire Coalfield

The succession, as seen in mining operations and at the nearest outcrops, consists of a variable series of arenaceous and argillaceous sediments with limestones in the lower part. Relatively thin basalt lava flows and small intrusions are seen in the Lower Carboniferous in Derbyshire, but the Clitheroe succession is free from these, and it is unlikely that similar igneous rocks occur in beds of this age in the area under consideration. The Coal Measures dip southward at about 10°, and in the south and west are unconformably overlain by gently dipping Trias. The area is cut by faults running mainly north-west to south-east, but their magnitude is not such that they are likely to have any magnetic effects. The proved thickness of the Middle Coal Measures is about 2750 ft. at Tyldesley near the Astley Green and Nook Collieries. The thickness of the Lower Coal Measures at outcrop is about 1000 ft. In contradistinction to the Middle Coal Measures the thickness of these beds is always very consistent, even when traced from one coalfield to another. Their thickness in the area under consideration is thus likely to be of the same order as that at outcrop.

At the beginning of Upper Carboniferous times the area under consideration was the site of a large delta bounded by land-masses occupying the Midlands and parts of North Wales and the southern Pennines. These land areas gradually subsided and the area of deposition extended, though the Midland Barrier persisted until the succeeding Coal Measures. Consequently as these former land-masses are approached, the lower beds of the Millstone Grit disappear and a complete succession is only to be expected where deposition was continuous from the Lower Carboniferous into Millstone Grit times. The present outcrop covers a considerable area of the southern Pennines and attains its greatest thickness in the mid-Pennine area, where it is of the order of 4500 ft. thick and conformable on the Lower Carboniferous. There is a thinning by alternation of the grit bands as the series is traced to the south, though the upper grits retain their thickness as far as North Derbyshire. Here the Millstone Grit is about 3000 ft. thick, but it appears that some of the lower beds are missing. To the west of the Pennines at Formby near Southport, a borehole proved over 1000 ft. of Millstone Grit of marine facies, the evidence suggesting that originally at least 6000 ft. were present, much having been removed by post-Carboniferous erosion (Kent 1948).

These and similar facts make it appear likely that the Lower Carboniferous land surface sloped down from the North Derbyshire area to a basin of deposition occupying the Lancashire and Cheshire plain. It is probable therefore that sedimentation was continuous here from the Lower Carboniferous into the Millstone Grit, and that the thickness of sediments deposited in the Nook and Astley Green region was at least as much as is met with in North Derbyshire; that is to say, it was at least 3000 ft. The beginning of the Lower Carboniferous period was marked by a gradual marine transgression. However, most of Wales and the Midlands remained a land area until well on into the Lower Carboniferous, and it seems possible that the land-mass stretched north as far as Derbyshire, since the oldest Lower Carboniferous deposits there do not seem to be much earlier than the zone of S_2 (Cope 1949) In North Wales the deposits are of the epi-continental sea type, the shoreline lying not fa to the south, as evinced by the fact that the basement beds occupy only the hollows in th Lower Palaeozoic synclines. In the early part of the Lower Carboniferous the sea was bounde to the north by the rigid Lower Palaeozoic block of north-west Yorkshire, the shorelin lying along the Craven Fault. South of the shoreline lay the deepest part of the sea in whic were deposited a great thickness of shales and thin limestones. In Derbyshire about 1500 ft. of limestones are exposed, and a further 900 ft. were proved in a boring near the crest of the Pennine anticline in Woo Dale (Cope 1949). Underlying the limestones were a series of green and purple volcanic rocks of presumed Pre-Cambrian age. In the mid-Pennine area south of the Craven Fault, the thickness of strata is of the order of 6000 ft. and the lowest beds are of Z age, indicating that deposition began much earlier than in Derbyshire. Deposits of similar facies to these were met beneath the Millstone Grit in the Formby borehole (Kent 1948), though here the indications are that the succession is somewhat thinner. There is, however, no certainty about this, as only part of the succession was passed through. It seems likely, therefore, that the subsiding Pre-Carboniferous floor shelved gently from the Derbyshire area to the north and west, and that it is justifiable to assume a thickness of Lower Carboniferous in the South-east Lancashire area at least as thick as that in Derbyshire, i.e. 2500 ft.

Table 2. Probable geological succession in South-East Lancashire

system	formation	thickness (ft.)
Pleistocene	Glacial sands and Boulder clay	100
Triassic	Bunter Sandstone	0
Carboniferous	Middle Coal Measures	2750
	Lower Coal Measures	1000
	Millstone Grit	3000
	Carboniferous Limestone Series	2500

The outcrops of Lower Palaeozoic and Devonian rocks lie at considerable distances from the South-east Lancashire Coalfield, and it is not possible therefore to make any inferences about their lithology or distribution underground in this area. This being so it seems reasonable to take 9000 ft., which, as is shown in table 2, is about the estimated depth of the base of the Carboniferous, as the limit above which extensive bodies of igneous rocks are unlikely to be present. In other words, no widely distributed rocks of susceptibilities high compared with those of the Carboniferous are likely to be met at smaller depths.

The South Yorkshire Coalfield

The Permian rocks, the scarp of which lies about a mile to the east of Hickleton Main Colliery, are dolomitic limestones underlain by sands. The Upper Carboniferous here is very similar to that of Lancashire. The Lower Carboniferous consists of limestone lying on sandstones and conglomerates, the latter of which may be in part Devonian. Lava flows, beds of ash and intrusive dolerites are met in the Lower Carboniferous of Derbyshire and are known from borehole data to occur in the Lower Carboniferous, Millstone Grit and Coal Measures in Nottinghamshire. The total thickness of igneous rocks is a few hundred feet. They appear to die out in the Upper Carboniferous to the north of Newark, and if present at all in South Yorkshire are likely to be confined to the Lower Carboniferous.

The area lies well to the east of the Pennine anticline, in the trough of a small shallow north-south syncline in the Middle Coal Measures. Consequently the strata here are almost horizontal, though the regional dip is to the east. In this direction the Coal Measures are unconformably overlain by the Permian magnesian limestones which dip in the same direction, though at a much smaller angle.

The Carboniferous is cut by two series of faults; one set trends north-west to south-east, and the other north-east to south-west. They have relatively small throws and are unimportant for the purposes of the experiments. The thickness of Coal Measures from the surface to the Parkgate seam, in which the observations were made, is about 2400 ft. Data from collieries to the west suggest that there may be a further 1600 ft. of Coal Measures before the base of the series is reached. There are therefore approximately 4000 ft. of Coal Measures present.

In North Derbyshire the Millstone Grit is approximately 3000 ft. thick, but it thins to the south and to the east. In South Derbyshire it is of the order of 1000 ft., and in borings at Eakring and Kelham Hills in Nottinghamshire it is 750 and 200 ft. thick respectively (Lees & Taitt 1946). From this and other borehole data it has been suggested that the facies and thickness trends for this series run north-north-east. Accepting this and extrapolating, an estimate of 1500 ft. is obtained for the thickness underground in Yorkshire.

TABLE 3. PROBABLE GEOLOGICAL SUCCESSION IN SOUTH YORKSHIRE

system	formation	thickness (ft.)
Permian (unconformity)	Magnesian Limestone	0
Carboniferous	Middle Coal Measures Lower Coal Measures	4000
	Millstone Grit	1500
Devonian	Carboniferous Limestone	3000
Lower Palaeozoic	possibly absent	—
Pre-Cambrian	—	—

There is a proved thickness of about 2500 ft. of Lower Carboniferous in Derbyshire. At Duke's Wood near Eakring, a borehole proved nearly 3000 ft. of sandstones and conglomerates (Lees & Taitt 1946), mostly Lower Carboniferous, but part of which may be Devonian. The South Yorkshire area lies well to the north of this borehole, away from the old Midland Barrier and to the east of the Pennine anticline. It is thus reasonable to suppose that in general the Lower Carboniferous in this area is perhaps 3000 ft. thick. The only place where thinning is likely to occur is on the crest of the Lindholme anticline, a concealed structure lying to the north of the area.

In the Foston borehole (Lees & Taitt 1946) possible Pre-Cambrian rocks were met below the Upper Palaeozoics, and in Derbyshire the Lower Carboniferous rests on supposed Pre-Cambrian. There is thus no geological evidence of the presence of Lower Palaeozoic rocks at depth in South Yorkshire. Since the Pre-Cambrian may consist of igneous rock the minimum thickness of sediments in this area, as shown in table 3, must be taken as 8500 ft. the sum of the thicknesses given for the Carboniferous and Devonian.

3·2. *The susceptibilities of the sediments and their resultant fields*

It is necessary to establish that the induced magnetism of the sedimentary rocks is insufficient to affect the measured values of ΔZ and ΔH. As the magnitude of the vertical component of the induced polarization is greater than that of the horizontal, the fields due to th latter are unlikely to exceed those due to the former. Thus if the likely anomaly in the vertical gradient can be shown to be negligible, the effect of the horizontal polarization need not be considered. For this purpose small specimens of the strata, taken at vertical intervals of about 30 ft., were obtained from boreholes and mine shafts in the various areas, and their susceptibilities measured to an accuracy of 10^{-6} units with an astatic magnetometer of the type described by Johnson & Steiner (1937).

Table 4. Rock susceptibilities

	rock type									
	mudstone		sandstone		fireclay		ironstone		all others	
locality	no. of samples	mean 10^6k	no. of samples	mean 10^6k	no. of samples	mean 10^6k	no. of samples	mean 10^6k	no. of samples	mean 10^6k
Lancashire:										
Astley Green Colliery	22	23	16	12	2	21	7	34	5	21
Culcheth Borehole	8	15	4	19	3	14	7	67	—	—
Burtonwood Borehole	10	21	1	12	2	18	—	—	1	55
all samples	40	21	21	13	7	17	14	51	—	—
Yorkshire:										
Rossington Colliery	6	19	5	12	2	10	3	69	2	28
Darrington Colliery	54	21	6	9	4	13	1	43	—	—
all samples	60	21	11	10	6	12	4	62	—	—
all samples for Lancashire and Yorkshire	100	21	32	12	13	15	18	53	—	—

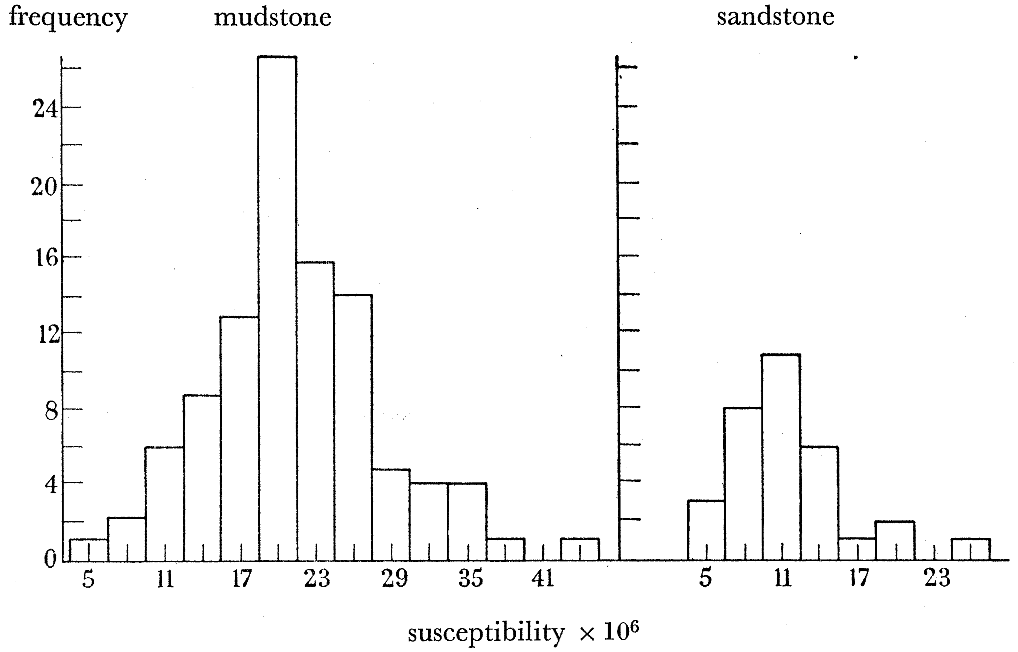


Figure 1. Frequency diagram of rock susceptibilities.

It was seen that the strata could be divided into four main lithological types, sandstones, shales, fireclays and ironstones, though the limits of the groups were somewhat arbitrarily defined. Table 4 gives for each locality the average value of the susceptibilities of samples of the different rock types, and figure 1 gives frequency diagrams of values of susceptibilities for two rock types. There is reasonably good agreement between the averages for corresponding rock types at all localities. This fact suggests that a representative sample of the strata has been taken, and that these averages can be used in computing the magnetic effects of the whole Coal Measure succession in the various localities without introducing serious errors. No systematic lateral or vertical distribution of susceptibility was observed in any area.

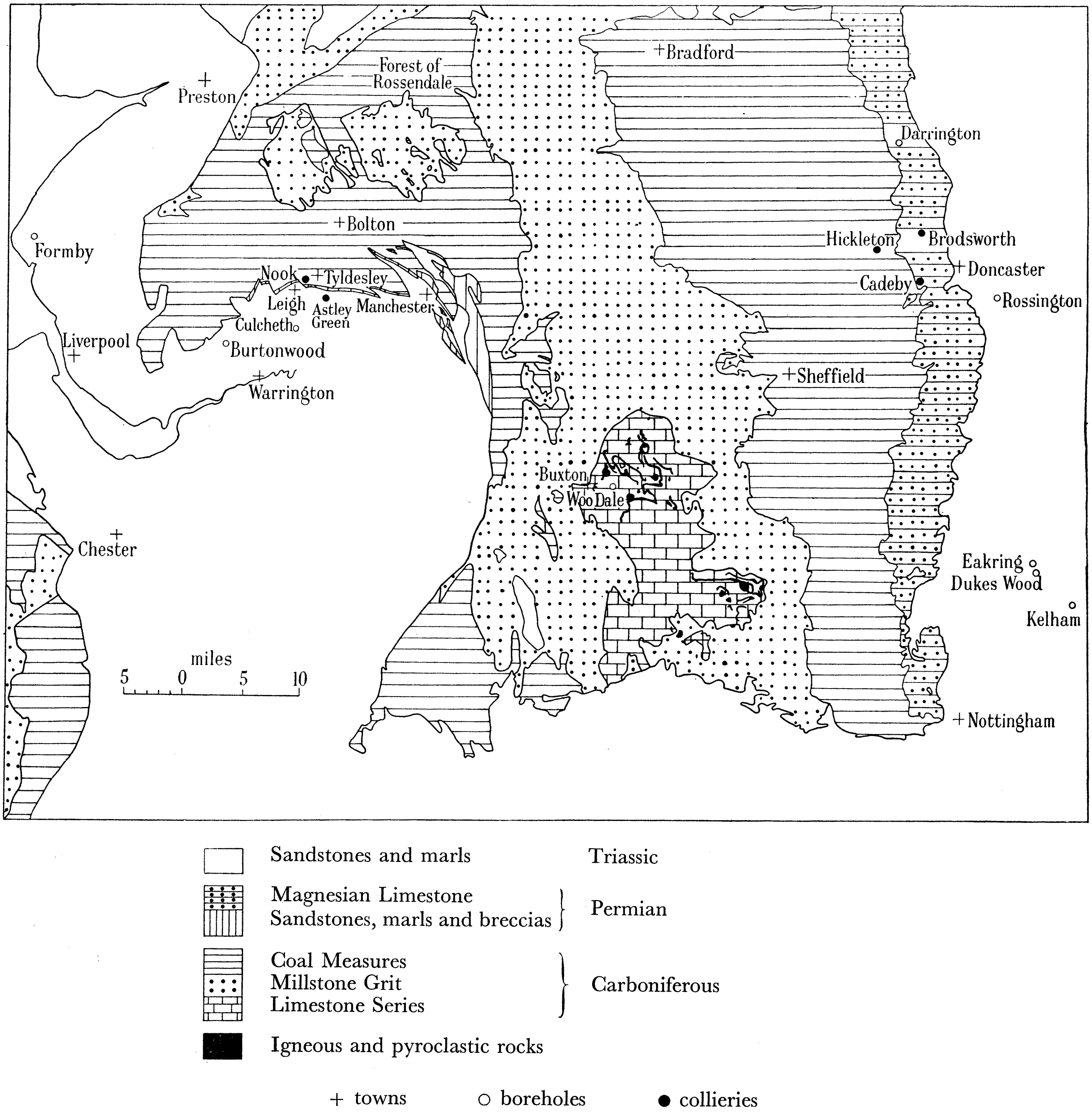


FIGURE 2*a*. Geological map of the North Midlands. Based on the $\frac{1}{4}$ in. to the mile map of H.M. Geological Survey.

The vertical sequence is known only at boreholes and mine shafts, perhaps a mile or so apart. In adjacent sections it is often possible to correlate coal seams and thick beds of shale and sandstone, but there is considerable lateral variation and impersistence in the thinner beds. The important facts apparent are that some beds persist over an area at least equal to that of a circle of about a mile radius, that the size distribution of beds is more or less random,

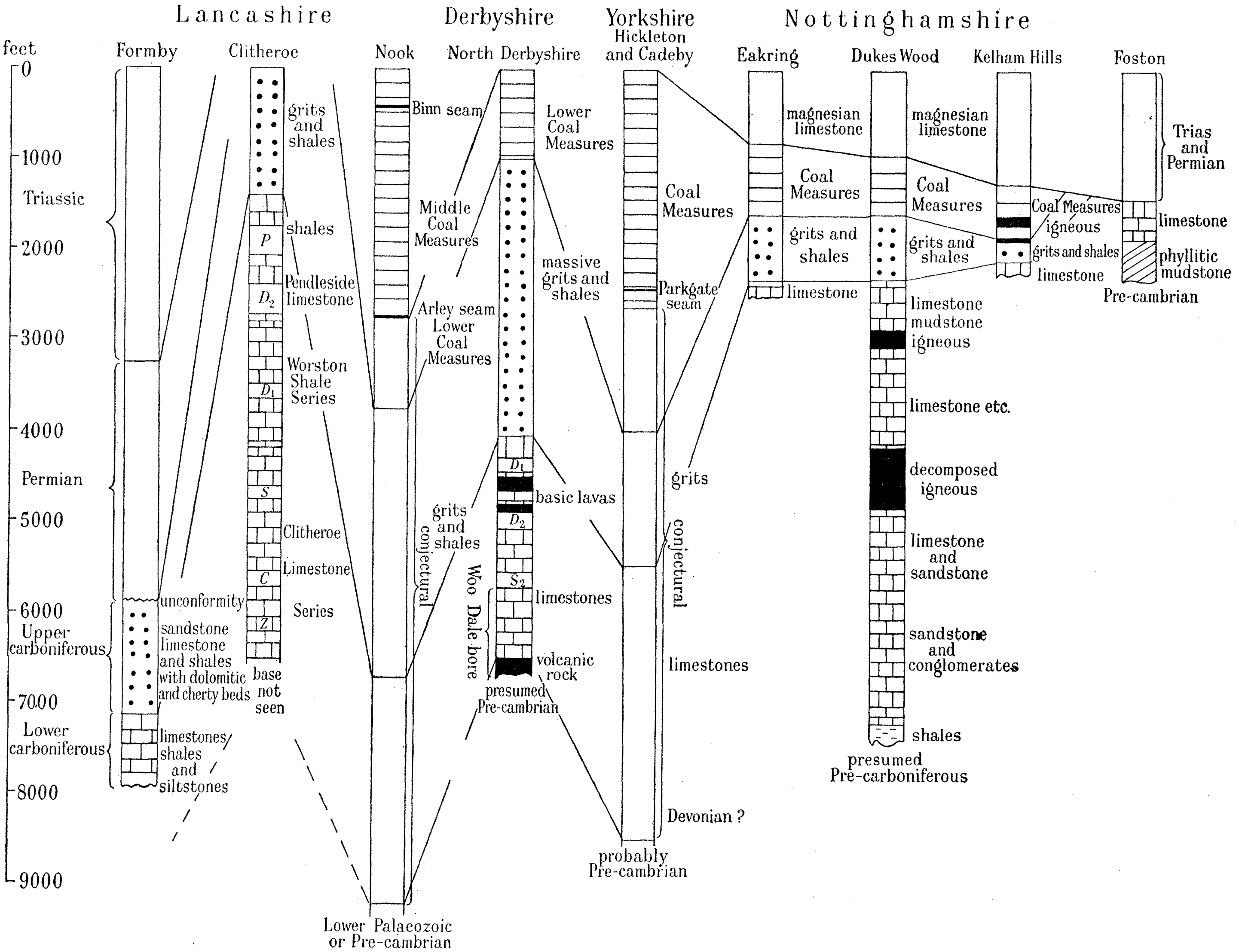


Figure 2*b*. Strata sections from boreholes and mines in the North Midlands with conjectures of the deeper structure.

and that at all points within such an area, the vertical succession, though differing widely in detail from point to point, will be substantially the same. Thus a sufficiently detailed knowledge of the strata is not available to allow the exact effect of the fields to be computed, but by making some reasonable simplifications approximate values can be obtained.

For the purposes of calculation we will consider each bed to be a horizontal disk of constant thickness (d) and radius (a), with constant susceptibility (k). Each such disk will produce, at a distance z along its axis, an anomaly in the vertical component of the earth's field DZ given by

$$DZ = 2\pi kZ\left[\frac{z}{(z^2+a^2)^{\frac{1}{2}}} - \frac{z+d}{[(z+d)^2+a^2]^{\frac{1}{2}}}\right]. \qquad (17)$$

DZ is greatest at a value of $a = \surd(2)\, z$ if $d \ll z$, giving a maximum value of

$$DZ = 2{\cdot}4\frac{kZd}{z}. \tag{18}$$

The difference between such maximum contributions to the vertical component at the surface and the underground bases, summed for all beds, is the maximum possible anomaly in Z, which will be very large compared with the actual one. However, this has been computed, and values for Yorkshire and Lancashire are given in tables 5 and 6.

TABLE 5. SOUTH YORKSHIRE: FINAL SUMMATION FOR VERTICAL COMPONENTS OF MAGNETIC FIELDS DUE TO STRATA

thickness of strata for which effect is summed (ft. from surface)	vertical components of magnetic fields in gammas					
	maximum effects			average effects		
	at surface	underground (2500 ft.)	difference	at surface	underground (2500 ft.)	difference
0 to 1183	+12	+ 2		+5	*	
1183 to 2728	+ 2	+25		*	+7	
2728 to 4000	0	+ 4		0	+4	
anomaly in ΔZ			+17			+6

* Not computed but less than 1γ.

TABLE 6. SOUTH-EAST LANCASHIRE: FINAL SUMMATION FOR VERTICAL COMPONENTS OF MAGNETIC FIELDS DUE TO STRATA

thickness of strata for which effect is summed (ft. from surface)	vertical components of magnetic fields in gammas					
	maximum effects			average effects		
	at surface	underground (2900 ft.)	difference	at surface	underground (2900 ft.)	difference
0 to 1428	+12	+ 2		−3	*	
1428 to 2656	+ 2	+25		*	−3	
2656 to 4000	0	+ 4		0	+4	
anomaly in ΔZ			+17			+4

* Not computed but less than 1γ.

A more realistic measure of the probable disturbance to the vertical field can be calculated by making the more reasonable assumption that the disks are randomly distributed in size up to a certain maximum radius R, the value of which was fixed at 5000 ft. from the geological considerations already discussed, though in actual practice the limits of a bed are not capable of exact definition. Then the average effect of one disk is given by the integration of equation (17):

$$DZ = -\frac{2\pi kZd}{R}\sinh^{-1}\frac{R}{z} \quad \text{if} \quad d \ll z. \tag{19}$$

The total effect of all beds was obtained, and these average anomalies in ΔZ are shown in tables 5 and 6. The effect of the Coal Measures between 2728 and 4000 ft. has been estimated, and the effect on the anomaly of all sediments below the Coal Measures is negligible.

The summation of the effect for all beds was made for the borehole or shaft section nearest to the experimental site, since the detailed succession was only known at such points. It was considered that the lateral variations in strata over the distance between the two would not cause any change in the result if it were possible to compute it for the section vertically below the experimental site.

A correction must also be made to the value of the field measured in the mine to allow for the induced and permanent magnetic polarization of the material surrounding the point of measurement. The determination of H was done, except in one case, in long passages nearly in the meridian. Thus the field measured was the true H. To the measured Z field a correction must be made, which is different according to whether the polarization of the surrounding material is induced or permanent. If the passage can be approximated by a long cylinder transverse to the field the correction E_3 to be added to the measured value is given by (20) for the case of induced magnetism and by (21) for the case of permanent magnetization:

$$E_3 \simeq +4\pi^2 k^2 Z, \tag{20}$$

where k is the susceptibility,

$$E_3 = -2\pi M, \tag{21}$$

where M is the polarization per unit volume.

From table 7 it will be seen that the corrections are negligible for all the mines.

Table 7. Susceptibilities and permanent polarization of samples from the mine roadways used as underground sites

locality	rock type	no. of samples	permanent polarization (10^{-6} gauss/cm.^{3})	mean 10^6k	correction to ΔZ (γ)
Astley Green Colliery	mudstone	8	0·3	15	0
Nook Colliery	siltstone	10	0·3	32	0
Hickleton Colliery	mudstone	6	0·3	25	0
Cadeby Colliery	sandstone	6	0·3	20	0
Brodsworth Colliery	sandstone	6	0·3	20	0

4. Instruments and technique

4·1. *General*

The measurements made in this experiment have been comparisons of the values of H and Z at different points, so that variometers rather than absolute instruments have been used. The accuracy required was 1γ, so that the instruments had to be set up sufficiently far from objects containing magnetic material (for example, 20 yards from a barbed wire fence), and observers had to be sure that they were not carrying any magnetic objects.

The readings of the moving variometers were corrected for the daily variation of the earth's field, which on quiet days consists of a smooth change over a range of some tens of gammas but on disturbed days is erratic and of larger amplitude. The daily variation is the same within a gamma over considerable distances, but it was determined by simultaneous readings of stationary instruments at the base rather than from observatory records. Figure 15 shows the correlation between the field changes at the surface and underground bases on a stormy day.

The sensitivity of each instrument was checked frequently in the course of the experiments by calibration fields from a Helmholtz coil system. The temperature coefficients of the instruments, all at least of the order of a few gammas per degree Centigrade, were likewise determined frequently over the temperature range encountered in the field. For these calibrations a non-magnetic hut was used with a specially designed variable temperature enclosure. An instrument in use must be at a constant temperature, and if subjected to a large temperature change about 30 min. must be allowed for it to attain the new ambient temperature. All the instruments require levelling to within 10 sec., and therefore it was necessary to set them up on firm ground.

4·2. *The variometers*

The H and Z variometers of the Lloyd Balance type (made by Hilger and Watts Ltd., London) were used in all the surface survey measurements because of their portability and the speed with which readings could be made. The H variometer consists of a vertical magnet mounted on a horizontal agate knife-edge balancing on horizontal quartz bearing-plates and set so that the system may rotate exactly in the magnetic meridian. The couple due to the horizontal field is balanced by a gravity couple, which is produced by offsetting the centre of gravity of the magnet system from the knife-edges by an amount which can be varied by a 'latitude screw', thus enabling the instrument to be used in different magnetic fields. The centre of gravity of the magnet system is raised by a 'sensitivity screw' until a change of the field of 1γ just produces a deflexion of the magnet system sufficiently large to be read on the scale of an autocollimating telescope. The moving system of the Z variometer is arranged so that its magnetic axis is horizontal. Fuller descriptions of such instruments are given by Heiland (1940).

To attain the accuracy desired, the variometers have to be protected from mechanical shocks during use, which tend to disturb the position of the centre of gravity of the magnetic system on which depends the value of the baseline (that is, the absolute value of the field corresponding to a fixed scale division). The knife-edges must be kept clean, otherwise the instruments fail to give the same reading after clamping and unclamping at the same station. This repetition was always tested by taking at least two successive readings. A change in the value of gravity alters the baseline of these variometers and a correction must be made. The changes are not important in surveying over limited areas at the surface, but g changes with depth by about 250 mgals every 4000 ft. in sediments of density 2·6 and produces a change of baseline of about 6γ. Large changes of temperature are liable to cause relative movements of the parts of the magnet system which disturb the baseline of the instruments. Thus they are unsuitable for accurate measurements of ΔZ and ΔH, in the course of which the instruments may be subjected to temperature changes of 30° C.

In the underground and surface surveys each moving instrument was read at the base before and after each set of measurements was made at the three field stations. This gives a closing error, which indicates a change of the baseline of the instrument. If the error was larger than 8γ the set was rejected, otherwise the closing error was distributed equally among the stations occupied. The closing error is the sum of the changes of baseline produced by each movement of the instrument between stations, all the changes being distributed about the value zero with a variance σ^2. Then in distributing the closing error equally between the stations occupied, the standard deviation of the resulting field values at the first and third stations will be $\sqrt{(3)}\sigma/2$ and at the second station σ. Thus, from the frequency diagrams of the closing errors of the survey, an experimental error can be attached to the determination of a field difference by the variometers. Figures 16 and 17 give such diagrams for the surface and underground surveys, the smaller closing errors in the latter being attributable to the smaller temperature fluctuations experienced in a mine and the smaller distances moved by the instruments in the course of the survey.

4·3. *The La Cour instruments*

For the comparison of H and Z between the underground and surface bases the Q.H.M. (quartz horizontal magnetometer) and the B.M.Z. (Z magnetometric balance), developed by the late Dr La Cour at the Danish Meteorological Institute, were used because of their great reliability. Not being absolute instruments they require to be standardized against observatory instruments, but their baselines remain constant to 1γ over many months. In particular, the stability of the Q.H.M. is very remarkable (Olsen 1942).

The Q.H.M., described in detail by La Cour (1936), consists of a horizontal magnet supported by a fine quartz fibre from the top of the vertical cylindrical case which encloses it. An autocollimating telescope is attached to the case through which the rotation of the magnet can be observed by a hair line reflected from a mirror on the magnet. The instrument is mounted on a divided circle which can be read to 10 sec. of arc. The magnet is set in the meridian and the tube is rotated to the west through an angle greater than one revolution until, by observation in the telescope, the magnet is reset in exactly the same position relative to the tube. In this position the fibre has been twisted through exactly 2π radians. This will be known as the $+2\pi$ position. A revolution in the opposite direction from the meridian is then made and is known as the -2π position. Half the difference (ϕ) between the readings of the divided circle with the temperature (t) enables H to be calculated to an accuracy of 1γ according to an empirical formula. Owing to the lack of damping each reading in one position takes about 5 min. in the field, in which time both H and the declination D may have altered. Thus another Q.H.M. was always used to record changes in D and a variometer used to record changes in H during each complete determination, so that a correction can be applied.

The B.M.Z., described in detail by La Cour (1942), has a magnet system as a field detector, the principle of which is similar to that of the Lloyd balance instruments, but it consists of a single piece of steel (called the Monad magnet) arranged to be exactly horizontal in zero field. It thus acts as a null detector and therefore changes in its magnetic moment and differences in gravity only affect the sensitivity and not the baseline of the instrument and are therefore unimportant. The major part of the earth's field is cancelled by a 'field magnet', which is situated at a fixed distance above the Monad magnet, and its temperature, which is kept uniform by lagging, is measured accurately by a thermometer within it. The field is then completely nulled by the small 'turn magnet', which may rotate about a horizontal axis at a fixed distance below the Monad magnet, its inclination to the vertical being read from a graduated circle. An autocollimating telescope enables the Monad magnet to be set in the horizontal position, which can be exactly determined by observing the deflexion of the magnet in the two positions when its axis is in the meridian. Only when the centre of gravity of the Monad magnet is vertically below the knife-edge does the telescope reading remain the same in both of these positions, for the sensitivity of the instrument is different in the two positions. The value of the field is obtained from an empirical formula containing the temperature of the field magnet and the position of the turn magnet.

5. The selection of underground sites

The induced intensity of magnetization I of a piece of iron or steel in a field H depends on the susceptibility k, on H and on the demagnetizing field H^1 (equal to NI), which arises from the induced poles. N is the demagnetizing coefficient depending on the shape of the body, values of which are given by Bozorth (1947). Thus

$$I = k(H-NI) = kH/(1+kN).$$

Since k is of the order of 20 for iron, kN is greater than 1 except for very elongated bodies, and therefore I is independent of k giving

$$I = H/N. \tag{22}$$

At distances great compared to its linear dimensions, a body of volume V produces a field approximating to that of a dipole of moment IV. Thus the effect of any mass of iron can be estimated, and as N is of the order of 1, the field due to the induced magnetism of 1 g. of iron at a distance of a metre is very roughly equal to $10^{-2}\gamma$.

There are only a few passages in mines which are sufficiently free from magnetic material and sufficiently remote from the large amount of steel in the vicinity of the pit shaft to be suitable for measurements of the earth's field. The shaft of a mine contains winding and guide ropes and usually a number of iron pipes, so that the effect at a point underground may be considered as due to a single pole at the bottom of the shaft. As an example the effect of the shafts at Hickleton may be considered, which contain pipes, ropes, etc., with a total area of cross-section of 900 cm.^{2}. This is a large figure, other shafts containing less iron. Using a value of 20 for the susceptibility and 0·4 gauss for the vertical field the pole strength of the shaft is 7000 gauss cm.^{2} which gives a negligible field at a distance of 900 ft. Surface plant is concentrated round the pit head and contains many iron and steel structures extending over about 100 yards. For instance, at Hickleton the field due to these structures is about 25γ at a distance of 1000 ft., as is known from an airborne survey (Jarman 1949); the field is therefore less than 1γ at an underground site 900 ft. from the shaft bottom at a depth of 2500 ft. Thus 900 ft. was taken as the minimum distance from a shaft at which an underground base could be established. At two collieries, Nook and Gresford, it was possible to confirm this estimate by a magnetic traverse on the surface. The effect at the surface site need not be considered as the site was arbitrarily chosen, and the value of the field measured there was corrected from the results of the surface survey.

Even passages free from steel structures were frequently found by a magnetic survey to have rails or other mining equipment buried in the stone debris forming the floor and walls, and these objects could often not be removed. Suitable sites were found in only fifteen collieries in this country, and of these only five satisfy the surface gradient criteria of § 2. These five collieries were: Nook and Astley Green, near Leigh, Lancashire; Hickleton Main, Cadeby and Brodsworth, near Doncaster, Yorkshire.

In order to examine the slight disturbances from the steel structures in neighbouring roadways, the H and Z fields in these passages were carefully surveyed and suitable points were selected as the underground bases. The results of these surveys are shown in figures 3 to 7, in which the abscissae are the distances in feet from arbitrary zeros near one end of each of the passages. The values of the closing error are distributed about zero with a standard deviation of $2{\cdot}0\gamma$ as shown in figure 17. Thus the standard error of the experimental determination of each value of the field in the underground passage is 1γ. In general, each survey shows a large gradient near to that end of the passage in which there are supports or which runs into a main roadway. This gradient decreases with distance from the roadway until the field becomes nearly constant. The absence of a gradient is not a complete guarantee that the steel at the ends of the passage no longer contributes to the field, for the gradient at a distance r from a line of dipoles is proportional to $1/r^3$ but the field strength is proportional to $1/r^2$, and a small gradient may be concealed by the local fluctuations of the field. The underground site was chosen to avoid areas where coal was being worked so that the steel in its neighbourhood was distributed only in the main roadways. These usually contained rails and arches, the amounts of steel being 100 and 20 lb. per foot of passage respectively.

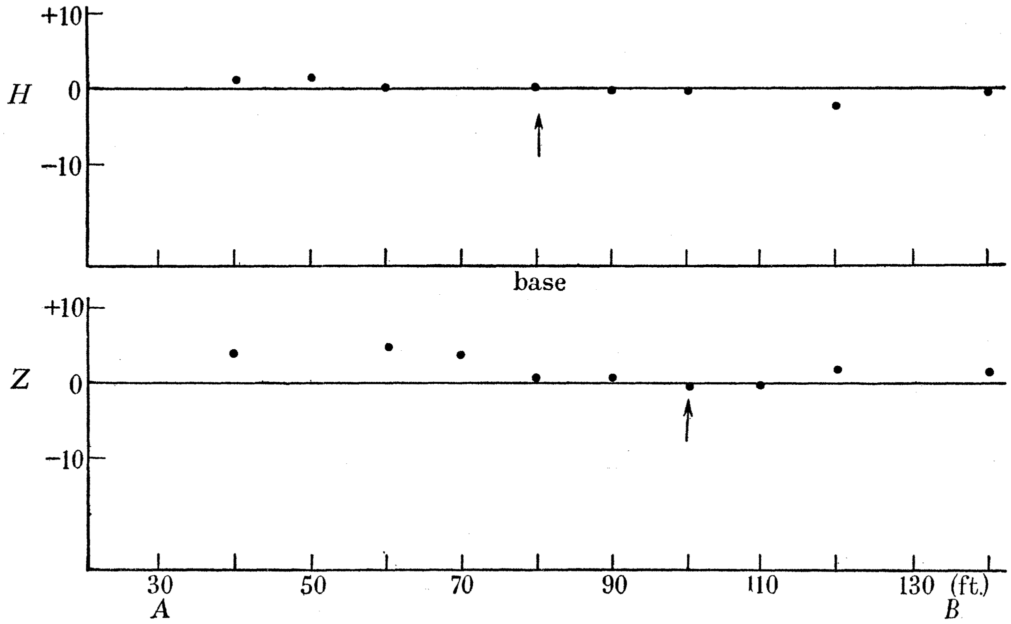


FIGURE 3. Astley Green: surveys of underground passage.

Three methods of estimating the disturbance of the field at the site have been considered. The effect of a concentration of iron of moment M at a distance l from the mid-point of a passage of length d was considered. If the vertical component changes linearly by an amount δZ over the length of the passage, the disturbance ϵ at the mid-point of the passage is less than δZ if $l \gg 3d$. A roadway supported by arches can be taken as equivalent to a line of vertical dipoles of moment m per unit length. If it is perpendicular to the base passage and at a distance l from its mid-point, then $\epsilon < \delta Z$ if $l \gg 2d$. Similar results can be found for other configurations and for the H component. However, these criteria cannot be applied when a line source is parallel to the base passage, and their practical use is limited by the fact that the small local variations in the field may make it difficult to determine δZ. Attempts to find the average value of the gradient by statistical fitting have been made, but the errors are large and the validity of the method is doubtful as it assumes random disturbance, whereas the disturbing iron is usually buried in the floor or pack wall and its effect is chiefly in one direction. The base passage at Cadeby is narrow and has solid walls, so that any buried

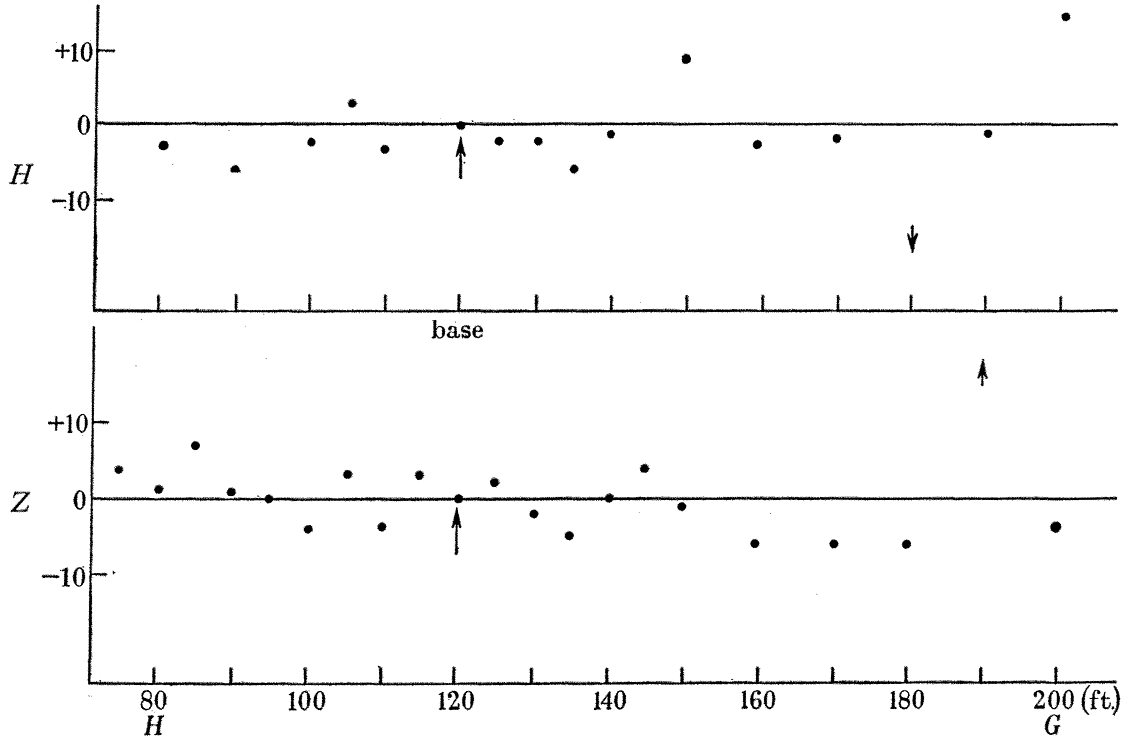


FIGURE 4. Nook: surveys of underground passage.

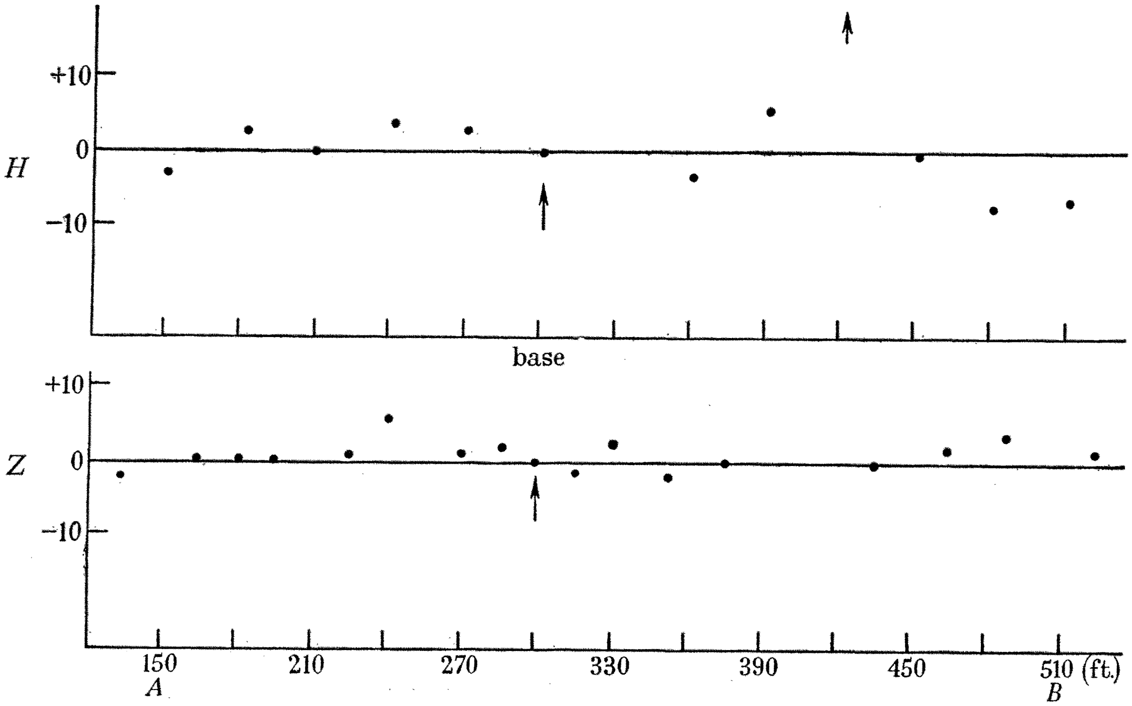


FIGURE 5. Hickleton: surveys of underground passage.

iron must be in the floor of the passage; thus the anomalies are largely positive and the undisturbed field of the passage is taken to be the lowest value of the field which most frequently occurred. The other site where the field is rather disturbed is at Nook, where the passage is wide and iron may be buried in both the walls and the floor; thus the average value of the field is taken as the undisturbed one. The second method was to make a survey along a line

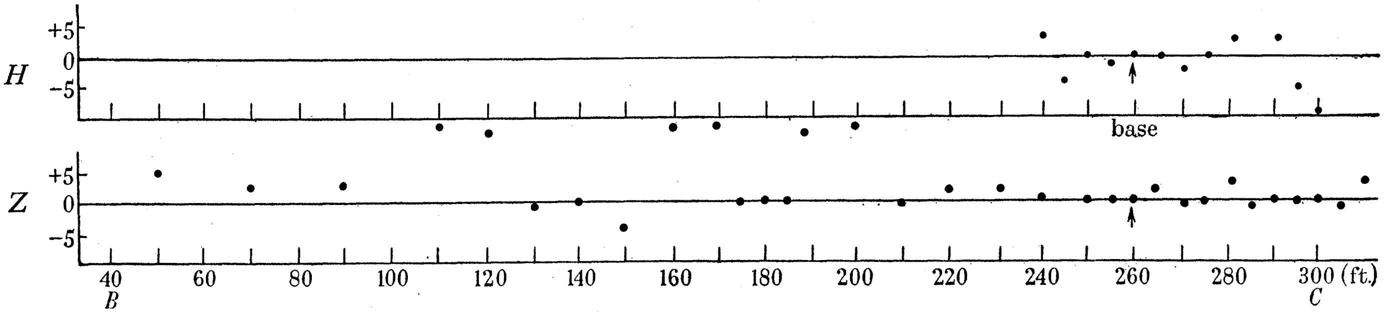


FIGURE 6. Cadeby: surveys of underground passage.

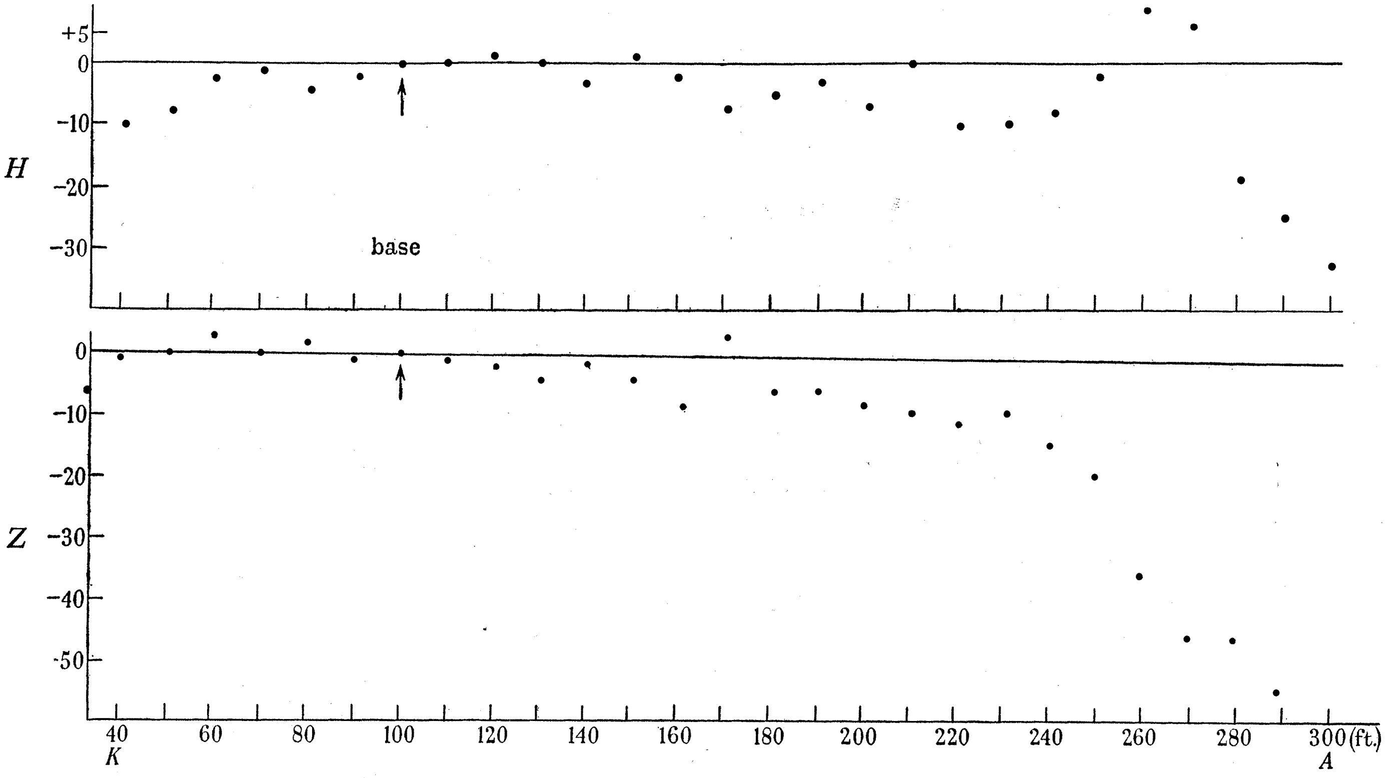


FIGURE 7. Brodsworth: surveys of underground passage.

up to the roadway and derive its effective moment from the gradient, where this is large enough to be estimated, as at Brodsworth. Thirdly, a rough order of magnitude of the disturbance field might be calculated from the volume and distribution of the steel, but this is very complicated, and it is not correct to assume that the magnetization is due to induction alone.

By the use of the second method at Brodsworth, estimates of the average vertical and horizontal moments per pound of steel have been derived. These results have been used to calculate the effects in other pits, as the rails and arches are of standard size. The results so obtained are regarded as a good approximation and, if small, can be applied as a correction to the measured values, but, if large, the site must be rejected. The situation of the base at Brodsworth Colliery is shown in figure 10 marked by a cross, and the results of the survey are shown in figure 7, the point 0 ft. being 335 ft. from the centre of the road *EAB*. An anomaly at about 260 ft. could not be removed without endangering the supporting pack wall, but it does not prevent smooth curves being drawn to show the fall-off of the field towards the roadway marked *EAB*; the parallel roads *MB* and *HE* do not affect this computation as they have a uniform effect along the passage. The contribution to the vertical field produced by the long passage *EAB* at a distance r from *A* along *AC* is $(w_1+w_2)\,m_z \sin\alpha/r^2$, where w_1 and w_2 are the weights of steel per unit length in the parts *AB* and *AE*, m_z the vertical moment per pound of steel, and α is the angle subtended at the point by *EA*. The value of Z in the region where the gradient is large (200 to 300 ft.) is plotted against the values of $\sin\alpha/r^2$. Such a plot should give a straight line of gradient $m_z(w_1+w_2)$. Figures 8 and 9 show that this is approximately the case, though the points for small values of r diverge, which is to be expected, since r is then of the same order of the breadth of the passage. Also the field at the base is depressed by the supports in passage *CK*, so that the points near the base do not lie on the lines of figures 8 and 9. The slope of the line in figure 8 is $(16\pm2)\times10^2$ gauss cm.^{2} and $w_1 = 4{\cdot}2$ lb./cm. and $w_2 = 0{\cdot}8$ lb./cm. Thus $m_z = (3{\cdot}2\pm0{\cdot}8)\ 10^2$ gauss cm.^{3}/lb. of iron.

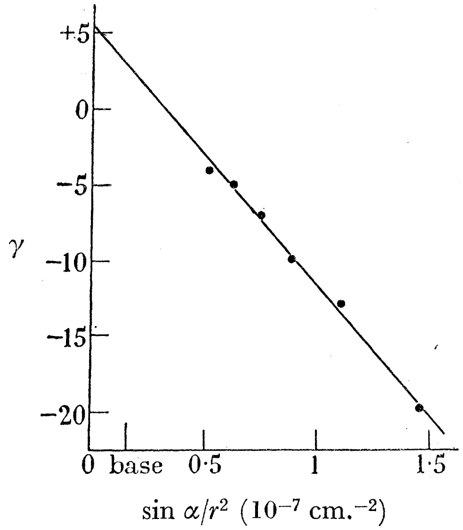


FIGURE 8. Brodsworth: analysis of *Z* survey.

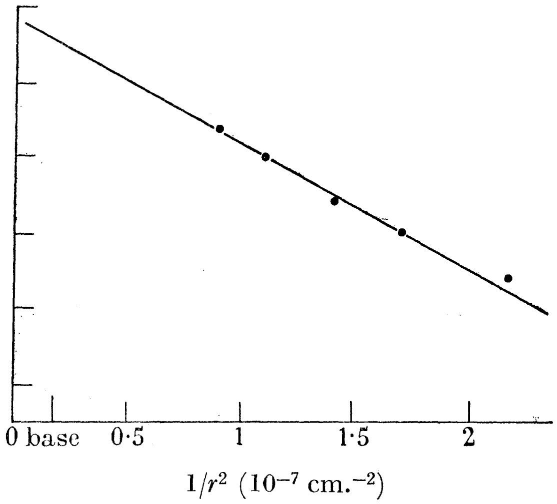


FIGURE 9. Brodsworth: analysis of *H* survey.

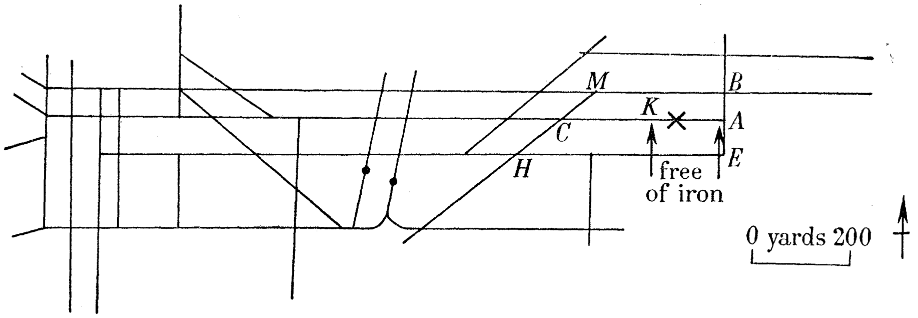


FIGURE 10. Brodsworth: plan of mine passages; —— roadways, × site, ● shafts.

With the H field anomaly, if the horizontal moment per pound of steel is m_H, the field in the passage AK is due to a line of dipoles in CK and to a pole at A of strength $m_H(w_1-w_2)$. Thus

$$m_H = (2{\cdot}4 \pm 0{\cdot}5)\ 10^2\,\text{gauss cm.}^3/\text{lb. of iron.}$$

As a working rule for the calculation of possible anomalies we take the Brodsworth results for m_Z and m_H, since the plots of Z and H against $1/r^2$ give reasonably straight lines. The polarizations m_Z and m_H will be proportional to the field components Z and H and to the demagnetizing coefficients. That $m_Z/m_H = 1{\cdot}3$ whilst $Z/H = 2{\cdot}5$ is probably due to the fact that the greater part of the arch is nearly vertical, the demagnetization coefficient being therefore greater in the vertical direction. The anomalies at each underground site due to those passages containing arches and rails, in each case a standard number per unit length, may now be calculated.

Brodsworth

The computed effects of the passages MB and HE at the base-point are $Z = -11\gamma$ and $H = +7\gamma$. From figures 8 and 9 the effect of passages EAB and CK is $Z = -6\gamma$ and $H = -4\gamma$. The anomaly in Z is large and of the same order as the expected effect ΔZ to be measured. This would preclude the use of the pit for the experiment were it not that the distribution of the neighbouring roadways is simple and the possibility of surveying up to the road EAB allows an estimate of the vertical anomaly which is probably trustworthy to within 25 %, i.e. $\pm 4\gamma$ in this case only. Further, though the estimate of the effect on H cannot be made so precisely, an over-estimate only gives a disturbance of $+5\gamma$. It therefore seems justifiable to include the results of Brodsworth.

Nook

The underground site is located on DE (figure 11) which is one of two disused haulage roads, DE and BC, leading to an abandoned district, from which all rails and arches had been removed. The site lies between the points H and G indicated by X. The nearest road containing steel is that marked BDF at 1200 ft. from the base which gives a negligible vertical field (about $0{\cdot}1\gamma$). The shaft is 1700 ft. away, and the surface traverse and the calculations above show that the effect of this will not be more than $0{\cdot}5\gamma$.

Astley Green

The base passage was driven into new rock away from the existing workings (figure 12). The shaft was 6000 ft. from the site, and the underground base was at the 80 ft. position. The field at the base point due to the steel supports at both ends of the site is $H = +1\gamma$ and $Z = -1\gamma$.

Hickleton

The location of the site is shown in figure 13, the shaft being 1600 ft. away. In addition to the supports in the roadways there were concentrations of girders at certain junctions, which were treated as single dipoles in the calculations. The base-point in the passage was at 300 ft., and the estimated anomalies here are $H = +1{\cdot}5\gamma$, $Z = -3\gamma$.

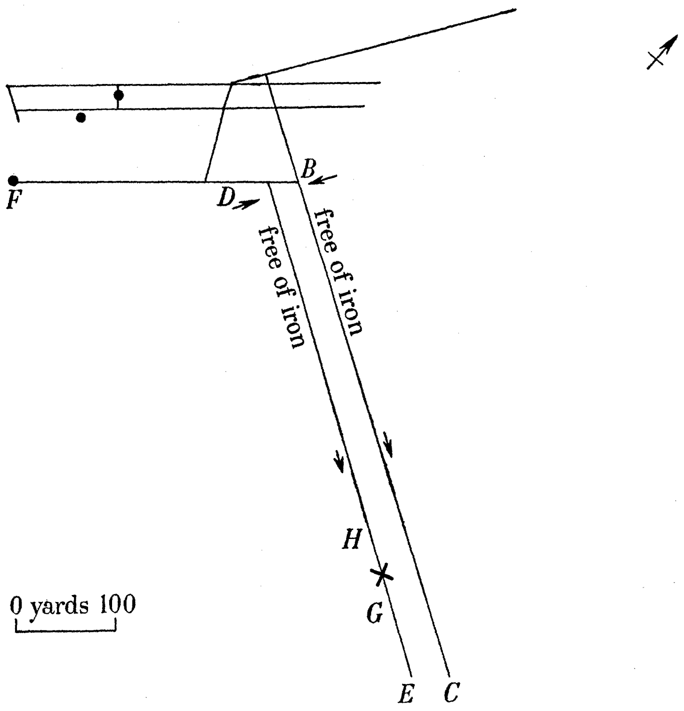


FIGURE 11. Nook: plan of underground passages; —— roadways, × site, ● shafts.

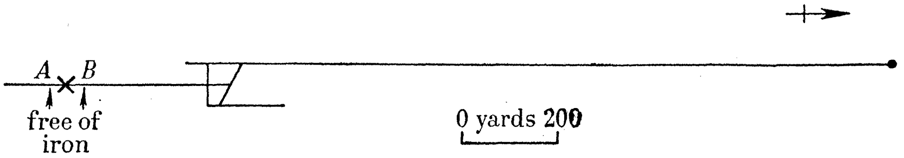


FIGURE 12. Astley Green: plan of mine passages; —— roadways, × site, ● shaft.

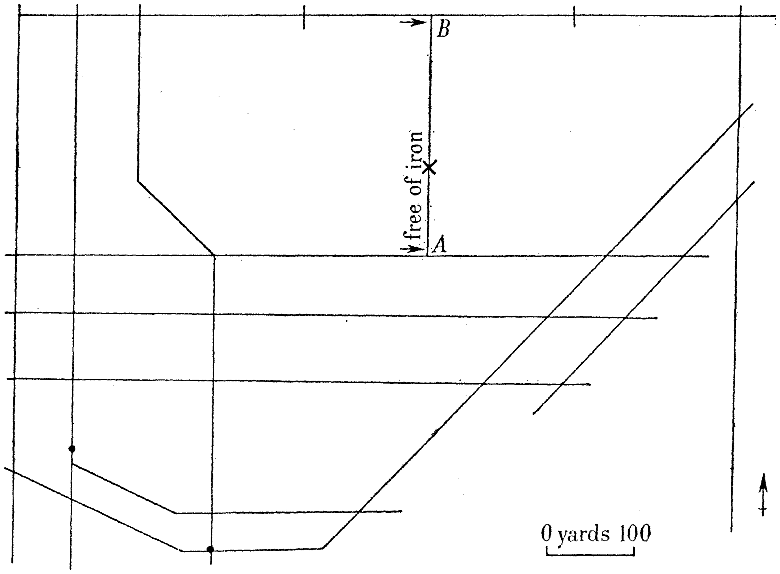


FIGURE 13. Hickleton: plan of mine passages; —— roadways, × site, ● shafts.

Cadeby

The location of the site is shown in figure 14, the shaft being 2500 ft. away. The only steel near enough to have a possible effect on the site was the rails in the passage AB. The base-point is at 260 ft. from the end of the rails at B and the calculated fields due to them are $H = +1\gamma$ and $Z = -1\gamma$.

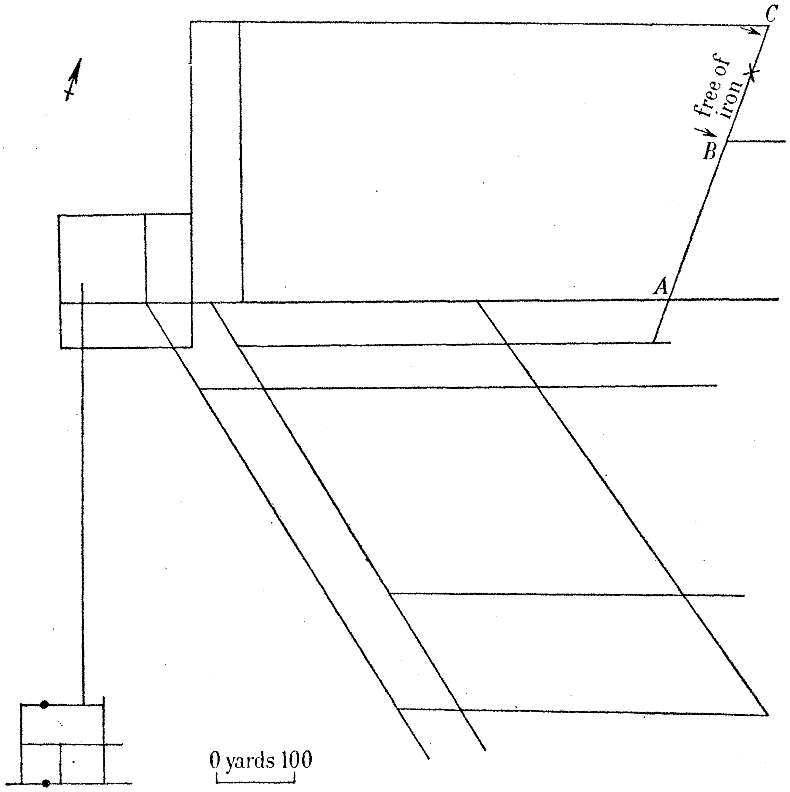


FIGURE 14. Cadeby: plan of underground passages; —— roadways, × site, ● shafts.

6. THE SURFACE SURVEYS

A magnetic survey on the surface in the region of the site of the experiment serves two purposes. It establishes the value of the field at the surface base in the absence of local fluctuations, by fitting linear expressions in x and y to a network of the values of H and Z. Secondly, the values of the gradients of the field components so determined are used to make corrections to the measured values of ΔZ and ΔH as explained in § 2.

Each survey consists of the determination of the relative values of H and Z at the intersections of the 1 km. national grid lines lying within a radius of 4 km. from the base-point. The values are referred to those at the surface base stations used for the ΔH and ΔZ measurements. The surveys were made using H and Z variometers in loops of three points starting from the base station and returning there to determine the closing error. At every point surveyed three observations were made at intervals of 10 ft. to guard against the possibility of the variometer being erected over buried iron.

Figure 16 shows for all instruments the frequency histogram of the closing errors obtained in all surveys from loops of three stations. The normality of the distribution may be tested by the method of moments described by Fisher (1950, chap. 3), in which statistics of the

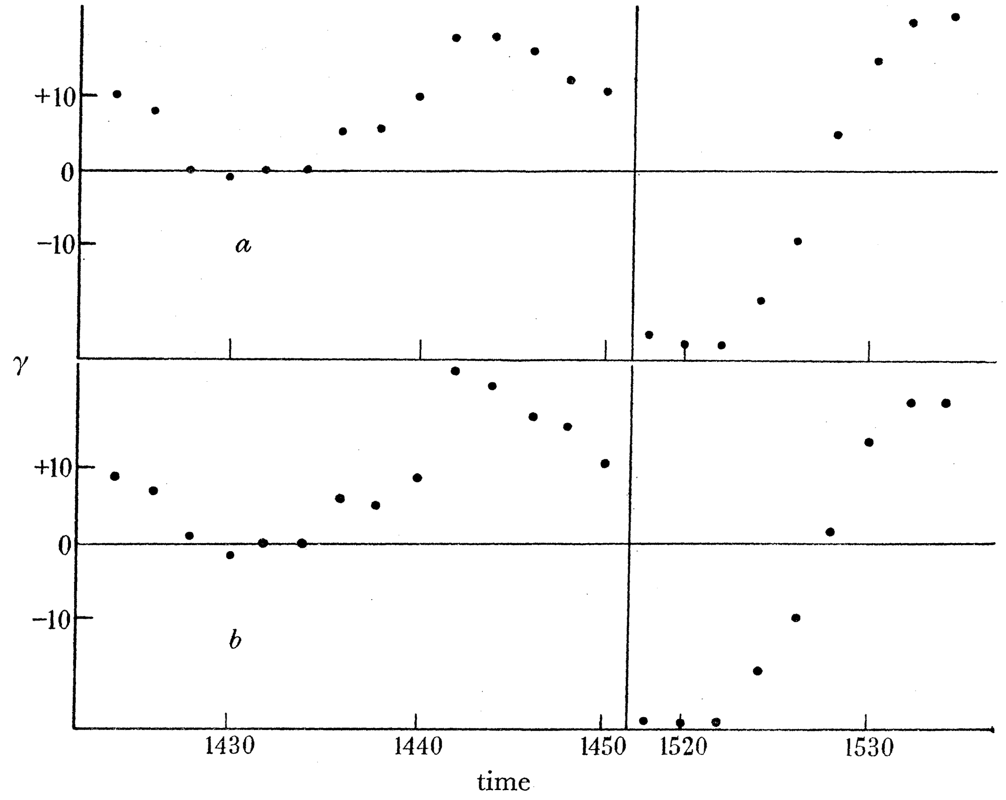
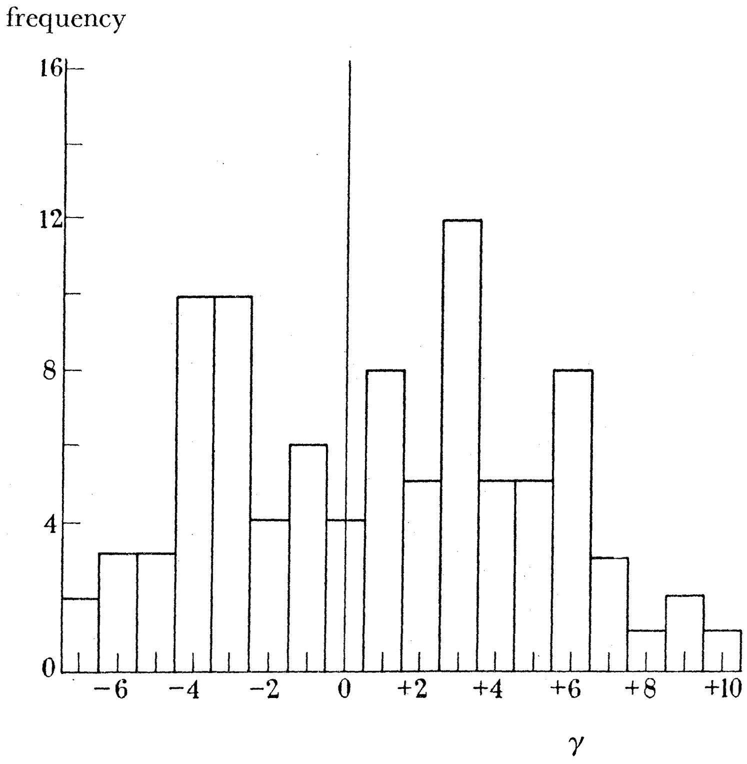


FIGURE 15. Daily variation of H on surface (a) and underground (b).



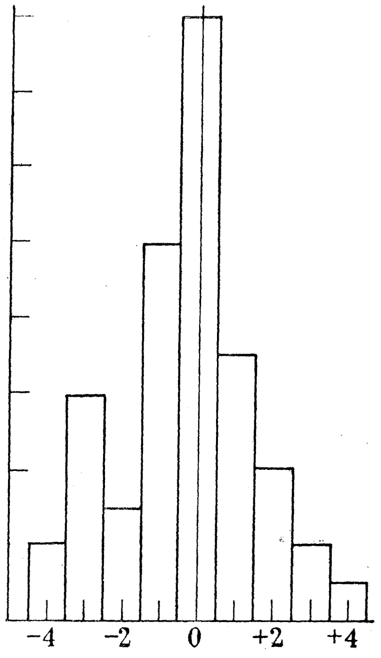


FIGURE 16. Closing errors of surface surveys.

FIGURE 17. Closing errors of underground surveys.

third and fourth degrees g_1 and g_2 respectively, estimates of parameters which are zero for a normal distribution, are compared against their sampling standard deviations. The values obtained are:

$$g_1 = -0{\cdot}20, \quad \text{S.D.} = \pm 0{\cdot}25,$$
$$g_2 = +0{\cdot}21, \quad \text{S.D.} = \pm 0{\cdot}50.$$

Thus neither g_1 and g_2 exceed their standard deviation and the distribution can be considered to be normal. The standard deviation of the closing errors is 4γ, giving a value of 2γ for the standard error of the experimental determination of each value of the survey network. The closing error was reduced to a minimum by guarding the instruments against shocks in transport. In order that these closing errors should not cause a systematic error in the value of a gradient, the three points are chosen at three corners of a square of 1 km. side formed by the grid lines. If more than three grid stations are occupied on any one loop, the closing error may become large, and if it was greater than 8γ the readings were rejected and the stations occupied once again.

The three mines in Yorkshire are covered by one survey (altogether consisting of 125 grid stations) and are referred to the base station at Brodsworth. In Lancashire a survey covering the areas of both collieries was made in 1949 and a further larger survey in 1950, the first sufficiently detailed to correct the surface base values of H and Z for local anomalies and the second consisting of 52 points to determine the field gradients. The points of the Yorkshire surveys were easy of access, the absence of villages made magnetic interference small, and the soil was solid enough to enable the instruments to be easily levelled. In Lancashire the presence of built-up areas and trolley-bus systems to the north of the area surveyed and boggy ground to the south caused the grid to be incomplete. It was found preferable for one observer to make all the observations with the moving instruments round a given loop. On the average six stations a day were occupied, and thus a minimum of 10 days was required to make a survey.

The sensitivities and temperature coefficients of the variometers were determined frequently during the surveys. Using these constants the observations round a single loop were reduced relative to the base stations, the closing errors being distributed equally among the stations. The results of the surveys in Yorkshire and in Lancashire are exhibited as plans in tables 8 *a* and *b* respectively. The values of Z and H, relative to those at the surface bases, are given beneath the six-figure grid references of the points to which the measurements refer. Regression equations in two variables x and y have been fitted to this network of values of H and Z, and the standard error of the coefficients have been computed (Fisher 1950). The Yorkshire survey has been treated in the above fashion and the Lancashire survey likewise but in two parts, since most of the area was surveyed twice owing to a change of base, with consistent results. The results of these computations are shown in table 9 and show that the gradients in Yorkshire and in Lancashire are nearly those to be expected from a dipole field ($dZ/dx = +5{\cdot}5\gamma$/km. and $dH/dx = -3{\cdot}5\gamma$/km.).

Figures 18, 19, 20 and 21 give specimen traverses in Z and H along north-south and east-west grid lines. Aerial traverses were made in a north-west to south-east direction over the Yorkshire site (Jarman 1949) using a fluxgate magnetometer measuring the variation in the total field intensity. The gradient was found to be $3{\cdot}0\gamma$/km., in good agreement with that of $2{\cdot}8\gamma$/km. predicted from the information of table 9. The standard deviation of the values

TABLE 8*a*. MAGNETIC SURVEY IN YORKSHIRE

The vertical field values are denoted by roman type.
The horizontal field values are denoted by italics.
The six-figure numbers give the grid references.

—	—	—	—	—	—	—	—	—	—	510110 +76 *−73*	—	519109 +79 *−72*	—	540110 +79 *−74*	550110 +80 *−74*	—
—	—	—	—	475101 +64 *−77*	485101 +71 *−76*	—	—	500100 +82 *−67*	—	510100 +63 *−64*	—	520100 +73 *−68*	530100 +64 *−70*	539101 +45 *−62*	551100 +72 *−69*	560100 +87 *−73*
—	445090 +65 *−69*	455090 +65 *−68*	465090 +67 *−65*	477090 +67 *−72*	485090 +61 —	—	495090 +65 *−78*	500090 +64 *−62*	—	509087 +63 *−61*	—	522090 +59 *−60*	530090 +64 *−63*	540090 +60 *−68*	550090 +58 *−61*	560090 +65 *−60*
—	445080 +60 *−66*	455080 +61 *−71*	465080 +61 *−68*	—	485080 +54 *−61*	490080 +56 *−55*	495080 +59 *−61*	499080 +54 *−62*	505080 +50 *−64*	511080 +68 *−66*	—	520082 +47 *−57*	—	—	550080 +60 *−55*	560080 +37 *−55*
437071 +50 *−64*	445070 +59 *−60*	455070 +61 *−62*	465070 +55 *−61*	—	485070 +57 *−53*	—	495070 +56 *−63*	500070 +49 *−52*	505070 — *−60*	510070 +54 *−49*	515070 — *−58*	521067 +41 *−41*	531070 +51 *−52*	540070 +48 *−53*	550070 +56 *−49*	560070 +52 *−50*
433057 +46 *−54*	445060 +50 *−61*	455062 +54 *−56*	—	—	485060 +50 *−55*	—	496060 +49 *−61*	500060 +43 *−51*	505060 +47 *−57*	510060 +42 *−47*	—	520060 +44 *−50*	530060 +43 *−39*	540060 +44 *−38*	—	560060 +31 *−39*
435050 +53 *−63*	445050 +42 *−62*	455050 +32 *−52*	465050 +31 *−51*	475050 +36 *−49*	485050 +45 *−53*	490051 +39 *−48*	495050 +43 *−52*	500051 +38 *−43*	505050 +40 *−51*	510050 +38 *−42*	515050 +41 *−48*	520050 +39 *−33*	529050 +40 *−45*	540052 +40 *−34*	—	—
435040 +32 *−50*	445040 +41 *−56*	—	467036 +41 *−53*	475040 +39 *−43*	485040 +36 *−46*	—	495040 +31 *−43*	500040 +35 *−19*	505040 +39 *−41*	510040 +29 *−41*	—	522039 +34 *−38*	530040 +30 *−35*	540040 +32 *−30*	—	—
—	445030 +21 *−54*	—	465030 +39 *−53*	473028 +22 *−51*	485030 +30 *−44*	490030 +32 *−23*	495030 +32 *−39*	500030 +33 *−17*	505030 +35 *−53*	511030 +29 *−33*	—	519030 +29 *−34*	531029 +28 *−32*	540030 +30 *−34*	—	—
—	—	455020 +29 *−56*	465021 +29 *−40*	475020 +20 *−44*	485020 +29 *−39*	490020 +25 *−33*	—	499020 +30 *−33*	—	513017 +20 —	—	520021 +17 *−28*	530020 +21 *−23*	—	—	—
—	—	—	—	—	480010 +21 *−28*	489010 +16 *−30*	—	499011 +18 *−26*	—	510008 +17 *−20*	—	520010 +13 *−21*	530010 +11 *−15*	540010 +19 *−21*	—	—
—	—	—	—	—	480990 +11 *−24*	490990 +7 *−20*	—	501990 −8 *−16*	—	510991 +14 *−2*	—	520990 −4 *−5*	530991 −8 *−4*	—	—	—

TABLE 8*b*. MAGNETIC SURVEY IN LANCASHIRE

(See table 8*a* for key.)

660040 +30 −*21*	—	678041 +16 −*13*	692038 +29 −*19*	699041 +29 −*23*	710040 +25 −*14*	—	—
—	—	—	690032 +29 −*13*	700030 +22 −*19*	710030 +24 −*16*	720030 +22 −*20*	—
—	670020 +3 −*7*	679018 +12 −*3*	—	—	—	720020 — −*17*	—
660011 +6 +*4*	670011 +10 +*6*	680010 +6 −*10*	—	700011 +11 −*8*	710010 +20 −*15*	—	—
—	671003 +9 +*5*	680000 +15 +*2*	690000 +15 −*1*	699000 −1 +*3*	710000 +13 −*11*	720000 — +*7*	728000 — −*10*
—	671990 −2 +*10*	680990 +3 +*5*	690990 +12 +*1*	700990 +16 −*3*	712990 +18 −*5*	721991 +20 +*12*	730990 +28 −*7*
660981 +15 +*16*	671981 +2 +*17*	680980 +14 +*6*	690980 +18 +*0*	700980 +19 −*1*	710980 +27 −*6*	720982 +21 −*2*	730980 +27 —
—	670970 +5 +*17*	680971 +13 +*13*	689971 +7 +*18*	700970 +16 +*10*	713969 +20 +*12*	720970 +20 +*2*	730970 +18 +*2*
—	667959 +3 +*30*	679961 +2 +*20*	691961 +7 +*15*	—	710960 +7 −	720960 +12 —	730960 +23 +*10*
—	670952 +2 +*33*	679948 +8 +*29*	—	—	—	720950 +16 —	—

TABLE 9. FIELD GRADIENTS

place	equation by partial regression	field at centroid	position of centroid	standard deviation of field at a point	no. of observations
Yorkshire	$Z=(-0{\cdot}6\pm0{\cdot}2)\,y+(6{\cdot}0\pm0{\cdot}2)\,x$	+43·3	45004054	5·5	124
	$H=(+0{\cdot}6\pm0{\cdot}2)\,y-(5{\cdot}0\pm0{\cdot}2)\,x$	−49·5	45004054	6·3	125
Lancashire, 1950	$Z=(+2{\cdot}5\pm0{\cdot}4)\,y+(1{\cdot}1\pm0{\cdot}3)\,x$	+15·0	36933995	6·0	52
	$H=(-1{\cdot}7\pm0{\cdot}4)\,y-(4{\cdot}5\pm0{\cdot}3)\,x$	+ 0·8	36923995	5·9	52
Lancashire, 1949	$Z=(+0{\cdot}8\pm1{\cdot}0)\,y+(1{\cdot}9\pm0{\cdot}6)\,x$	+18·0	36973996	—	25
	$H=(-3{\cdot}2\pm0{\cdot}8)\,y-(4{\cdot}4\pm0{\cdot}5)\,x$	− 6·0	37004000	—	25

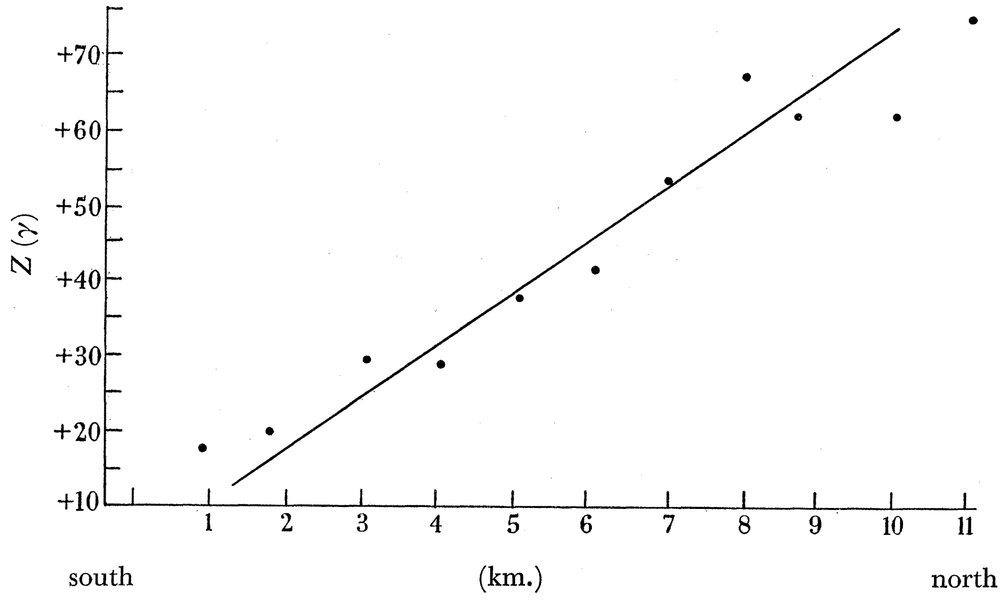


FIGURE 18. Magnetic traverse, gridline 451 (Yorkshire).

of Z and H at a point in the survey from the value given by the linear regression formula is given in the fifth column of table 9. This is considerably larger than the experimental error of the instruments used; a part may result from the non-linear gradients and part may be accounted for by irregularities of the magnetic field due to local causes. Much of northern England is covered by glacial drift, in which there is likely to be igneous material with a magnetite content higher than that of the sediments.

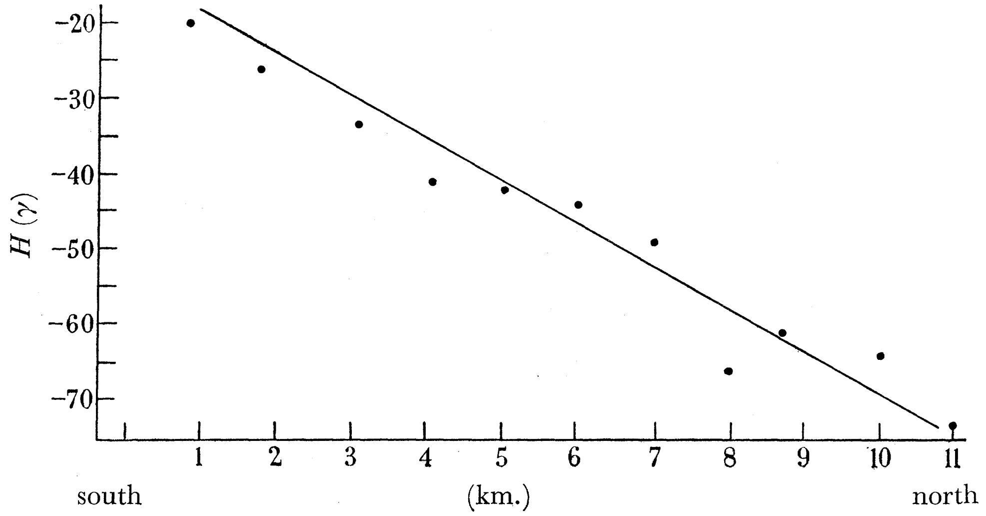


FIGURE 19. Magnetic traverse, gridline 451 (Yorkshire).

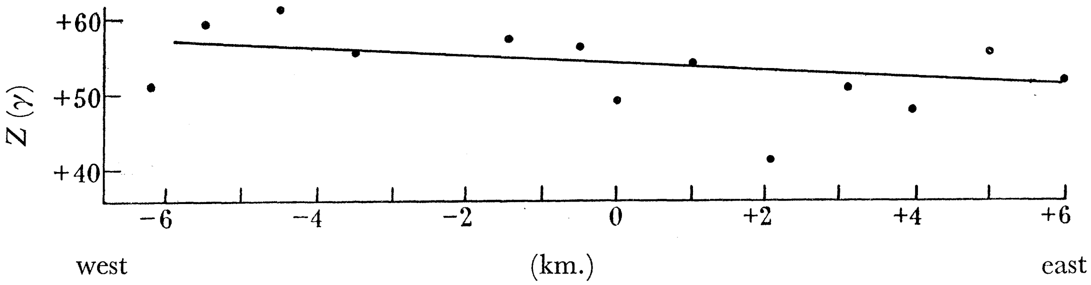


FIGURE 20. Magnetic traverse, gridline 407 (Yorkshire).

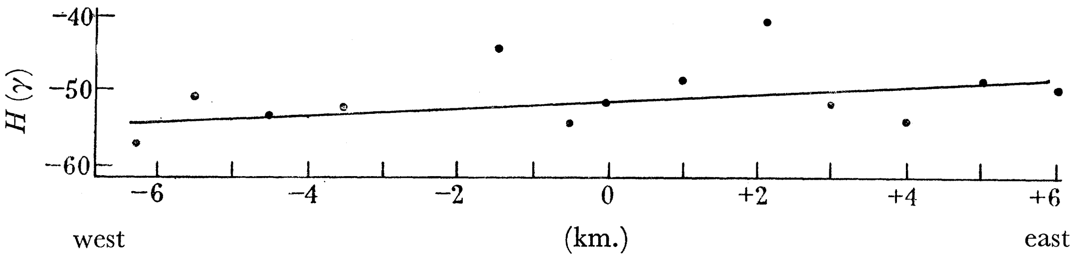


FIGURE 21. Magnetic traverse, gridline 407 (Yorkshire).

Information concerning the non-linearity of the gradients can be obtained by fitting parabolic laws of the type

$$Z = A + Bx + Cx^2$$

to the traverses. This fitting was performed for each line of latitude and longitude for each component. The results are shown in table 10 and are seen to amount to less than a gamma over a distance equal to the depth of the mines (0·8 km.).

TABLE 10

Lancashire H

north to south traverses									
eastings	730	720	710	700	690	680	670	660	
coefficient of x^2 (γ/km^2.)	-0.8	$+2.0$	-0.9	$+0.2$	-0.2	-0.3	-0.2	$+0.9$	
east to west traverses									
northings	040	030	020	010	000	990	980	970	960
coefficient of y^2 (γ/km^2.)	$+0.1$	-0.5	$+1.6$	-0.3	0.0	-0.5	-0.6	$+0.4$	-0.7

Lancashire Z

north to south traverses								
eastings	730	720	710	700	690	680	670	660
coefficient of x^2 (γ/km^2.)	-1.5	$+0.1$	$+0.1$	-1.2	-0.1	0.0	-0.2	-1.8
east to west traverses								
northings	040	030	010	000	990	980	970	960
coefficient of y^2 (γ/km^2.)	-1.2	-1.2	-1.6	-2.5	$+0.2$	0.0	$+0.4$	-0.2

Yorkshire H

north to south traverses														
eastings	560	550	540	530	520	510	500	495	485	475	465	455	445	435
coefficient of x^2 (γ/km^2.)	$+0.9$	$+0.3$	$+0.2$	0.0	0.0	-0.1	0.0	0.0	-0.1	$+0.5$	$+0.2$	$+0.6$	0.0	$+0.6$
east to west traverses														
northings	110	100	090	080	070	060	050	040	030	020	010	099		
coefficient of y^2 (γ/km^2.)	-12.6	$+0.2$	-0.2	0.0	0.0	-0.5	0.0	-3.1	$+0.1$	$+0.3$	$+0.4$	$+0.7$		

Yorkshire Z

north to south traverses														
eastings	560	550	540	530	520	510	500	495	485	475	465	455	445	435
coefficient of x^2 (γ/km^2.)	-1.3	-3.0	0.0	$+0.2$	0.0	-0.1	$+0.2$	$+0.1$	-0.5	$+0.1$	-0.9	-0.5	$+0.9$	-0.7
east to west traverses														
northings	110	100	090	080	070	060	050	040	030	020	010	099		
coefficient of y^2 (γ/km^2.)	-0.7	-0.2	0.0	$+0.1$	0.0	-0.5	-0.8	$+0.1$	$+0.3$	$+0.1$	$+2.0$	-1.8		

TABLE 11

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
									Z variometer no. 41						
position	time (B.S.T.)	turn magnet reading (degrees)	magnet Z_T field	temp. (°C) T	αT	temp. gradient $\dot{T}$ (°C/min.)	correction for $\dot{T}$	Z	temp. (°C)	reading (divisions)	KT	dZ	$Z-dZ$	mean $Z-dZ$	ΔZ
Brodsworth surface	1356	72·25	346	16·01	+256	−0·01	0	44383	16·7	19·2	0	+ 0	44383		
	1357	72·2	347	16·01	+256	0	0	44384	16·7	19·2	0	+ 0	44384		
	1358	72·2	347	16·00	+256	+0·01	0	44384	16·7	19·2	0	+ 0	44384	44384	
	1359	72·2	347	16·02	+256	0	0	44384	16·7	19·2	0	+ 0	44384		
	1400	72·2	347	16·03	+256	0	0	44384	16·7	19·2	0	+ 0	44384		
Brodsworth underground 101 ft.	1504	56·65	626	30·24	+484	+0·01	0	44435	17·4	19·65	+2	+ 7	44428		
	1506	56·6	627	30·25	+484	+0·01	0	44436	17·7	19·45	+3	+ 2	44434		
	1508	56·6	627	30·28	+484	+0·01	0	44436	17·9	19·55	+4	+ 3	44433		
	1510	56·5	628	30·30	+485	+0·02	0	44436	18·0	19·8	+4	+ 6	44430		+45
	1512	56·5	628	30·35	+486	+0·01	0	44435	18·0	19·8	+4	+ 6	44429		
	1514	56·5	628	30·36	+486	+0·01	0	44435	18·2	19·8	+5	+ 7	44428		
	1516	56·55	628	30·37	+486	0	0	44435	18·4	19·85	+6	+ 7	44428		
	1518	56·5	628	30·37	+486	0	0	44435	18·6	19·85	+6	+ 7	44428		
	1520	56·5	628	30·37	+486	0	0	44435	18·7	19·9	+6	+ 8	44427		
	1522	56·5	628	30·38	+486	0	0	44435	18·8	19·9	+7	+ 7	44428	44429	
	1524	56·4	630	30·37	+486	0	0	44437	18·8	19·9	+7	+ 7	44430		
	1526	56·45	629	30·37	+486	0	0	44436	18·9	19·95	+7	+ 8	44428		
	1528	56·4	630	30·37	+486	0	0	44437	18·9	19·95	+7	+ 8	44429		
	1530	56·4	630	30·37	+486	0	0	44437	18·9	19·95	+7	+ 8	44429		
	1532	56·45	629	30·37	+486	0	0	44436	18·9	20·0	+7	+ 9	44427		
	1534	56·4	630	30·37	+486	0	0	44437	18·9	20·0	+7	+ 9	44428		
Brodsworth surface	1720	70·95	371	15·65	+250	0	0	44414	17·0	20·85	+1	+32	44382		+46
	1722	70·95	371	15·65	+250	0	0	44414	17·0	20·65	+1	+28	44386		
	1723	71·0	370	15·63	+250	−0·02	0	44413	17·0	20·75	+1	+30	44383		
	1724	71·0	370	15·62	+250	0	0	44413	17·0	20·7	+1	+29	44384		
	1725	71·0	370	15·61	+250	−0·01	0	44413	17·0	20·75	+1	+30	44383		
	1727	71·1	368	15·59	+249	−0·02	0	44412	17·0	20·65	+1	+28	44384		
	1728	71·1	368	15·57	+249	−0·01	0	44412	17·0	20·75	+1	+30	44382		
	1730	71·15	367	15·55	+249	−0·02	0	44411	17·0	20·7	+1	+29	44382	44383	
	1731	71·15	367	15·53	+248	−0·01	0	44412	17·0	20·65	+1	+28	44384		
	1732	71·15	367	15·52	+248	0	0	44412	17·0	20·7	+1	+29	44383		
	1733	71·15	367	15·51	+248	0	0	44412	17·0	20·75	+1	+30	44382		
	1734	71·15	367	15·51	+248	0	0	44412	17·0	20·75	+1	+30	44382		
	1735	71·15	367	15·50	+248	0	0	44412	17·0	20·75	+1	+30	44382		

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

1	2	3	4	5	6	7	8	
			Q.H.M. 88 as declinometer			Q.H.M. 89		
			micrometers				micrometers	
place	time (B.S.T.)	position	*A*	*B*	mean *dD*	temp. (°C)	*A*	
odsworth surface	0947	0	238° 34′ 59″	058° 24′ 58″	00″		160° 14′ 32″	340
	0949	$+2\pi$	238 34 52	58 24 50	− 07	16·3	113 52 26	294
	1001	-2π	238 34 01	58 24 02	− 57	16·4	206 31 48	026
	1007	0	238 34 12	58 24 11	− 47		160 13 28	340
	1007	0	238 34 12	58 24 11	00		160 13 28	340
	1014	$+2\pi$	238 34 00	58 23 55	− 14	16·7	113 51 03	294
	1021	-2π	238 33 35	58 23 35	− 36	17·0	206 32 30	026
	1028	0	238 33 35	58 23 35	− 36		160 12 56	340
odsworth underground, 101 ft.	1242	0	—	62 16 48	00		173 12 53	353
	1250	$+2\pi$	—	62 16 58	+ 10	30·3	126 20 42	306
	1256	-2π	—	62 16 28	− 20	30·4	219 58 37	040
	1304	0	—	62 16 29	− 19		173 12 00	353
	1310	0	—	62 16 30	00		173 12 01	353
	1314	$+2\pi$	—	62 15 57	− 33	30·4	126 21 32	306
	1320	-2π	—	62 16 14	− 16	30·4	219 57 30	040
	1329	0	—	62 16 30	00		173 12 03	353
odsworth surface	1616	0	243 05 47	62 55 42	00		186 48 18	007
	1620	$+2\pi$	243 05 20	62 55 17	− 26	19·6	140 24 17	320
	1628	-2π	243 04 40	62 54 38	−1′ 05	20·6	233 06 02	053
	1633	0	243 04 39	62 54 34	−1′ 08		186 47 18	007
	1633	0	243 04 39	62 54 34	00		186 47 18	007
	1643	$+2\pi$	243 04 55	62 54 51	+ 16	21·4	140 25 42	321
	1656	-2π	243 05 10	62 55 10	+ 33	21·6	233 03 48	053
	1701	0	243 05 00	62 55 03	+ 25		186 48 00	007

TABLE 12

9	10	11	12	13	14	15	16	17
rs							variometer reading $H12$	
B	mean $-dD$	ϕ	log sin ϕ	mean KT	log H	H	temp. (°C)	(divisions)
0° 29′ 37″ 4 07 40 6 47 07 0 28 50	00″ 46° 21′ 54 46 18 20 −08	46° 20′ 08″	9·859376	0·002918	9·242218	17467	18·2 17·0	23·7 23·5
0 28 50 4 06 26 6 47 48 0 28 22	00 46 22 10 46 19 36 +06	46 20 53	9·859461	0·003008	9·242223	17467	16·8 16·9	23·6 23·6
3 28 42 6 36 14 0 14 08 3 27 52	00 46 52 28 46 45 55 −32	46 49 12	9·862851	0·005417	9·241242	17428	17·0 17·0	23·2 23·4
3 27 52 6 37 03 0 13 17 3 27 40	00 46 50 11 46 45 43 −05	46 47 57	9·862698	0·005426	9·241404	17434	17·0 17·0	23·9 23·6
7 03 18 0 39 41 3 21 15 7 02 30	00 46 23 23 46 18 55 +14	46 21 09	9·859500	0·003587	9·242763	17489	19·5 20·0	24·4 25·3
7 22 30 1 00 56 3 39 04 7 03 22	00 46 21 15 46 15 59 +22	46 18 55	9·859230	0·003837	9·243283	17510	20·2 20·8	25·7 26·4

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17	18	19	20	21	22	23
meter g H12 (divisions)	KT	dH	$\overline{dH}$	$H-\overline{dH}$	$\overline{H}$	ΔH
23·7	−1	+1	−1	17468		
23·5	0	−3				
					17468	
23·6	0	−1	−1	17468		
23·6	0	−1				
						−34
23·2	0	−7	−6	17434		
23·4	0	−5				
					17434	
23·9	0	+3	0	17434		
23·6	0	−2				
						−34
24·4	−2	+14	+21	17468		
25·3	−2	+28				
					17468	
25·7	−3	+35	+42	17468		
26·4	−3	+48				

(*Facing p.* 144)

7. The determination of ΔH and ΔZ

For the measurement of ΔZ the B.M.Z. was set up at the surface base with a Z variometer to record the daily variation dZ, the latter being sheltered by a tent. About ten simultaneous readings of the B.M.Z. and variometer were taken, the turn magnet being reset to the null position for each reading. Between these settings the B.M.Z. thermometer was read at intervals to give values of the temperature and its rate of change. The B.M.Z. was then taken into the mine, where about twenty readings were taken simultaneously with readings of the variometer on the surface. The B.M.Z. was then returned to the surface base and a third set of readings were made.

A measurement of ΔH usually occupied a whole day. The Q.H.M., the declinometer and an H variometer were initially set up in tents at the surface base, and at least two sets of readings of the Q.H.M. were taken. The Q.H.M. and declinometer were then taken underground and the procedure repeated. The variometer on the surface and a second one underground were read at the same time as the $+2\pi$ and -2π readings of the Q.H.M. The averages of the readings of the surface and underground variometers were used to eliminate errors due to slight differences in timing and in the response times of the instruments. The instruments were then brought to the surface and a third set of readings taken.

Examples of the reduction of a complete determination of ΔZ and ΔH using the B.M.Z. and the Q.H.M. are given in tables 11 and 12 respectively. The vertical component (Z) of the field as measured by the B.M.Z. is given by the reduction formula

$$Z = Z_1 + Z_T - \alpha T - 25\dot{T}, \tag{23}$$

where T is the temperature of the instrument, α its temperature coefficient, Z_1 a constant and Z_T the vertical component of the field of the turn magnet at the Monad magnet, which is obtained from the calibration tables of the turn magnet dial. The last term ($25\dot{T}$) is a small correction which is applied should the temperature be changing *slowly* at a rate $\dot{T}$ degrees Centigrade per minute. Table 11 records the determination of ΔZ at Brodsworth on 4 July 1950. Five simultaneous readings of the B.M.Z. no. 53 and the Z variometer no. 41 were taken at the surface, sixteen simultaneous readings with the B.M.Z. underground and a further thirteen readings with the B.M.Z. returned to the surface. Columns 3, 5 and 7 contain the B.M.Z. observations and columns 4, 6, 8 and 9 their reduction. Columns 10 and 11 contain the Z variometer readings and columns 12 and 13 their reduction, the sensitivity of Z41 variometer being 19·2 γ/scale division and its temperature coefficient (K) +3·1 γ/° C. Column 14 gives the value of Z determined by the B.M.Z. corrected for the daily variation. The average field at the various sites is given in column 15 and ΔZ in column 16. The horizontal component (H) of the field as measured by the Q.H.M. 89 is given by the reduction formula

$$\log_{10} H = 9{\cdot}09870 - \log_{10}\sin\phi + 0{\cdot}0001785\,T - 0{\cdot}0002H\cos\phi, \tag{24}$$

where 2ϕ is the difference of the divided circle micrometer readings in the $+2\pi$ and -2π positions, corrected for the change of declination dD in the time between these readings. Table 12 shows a determination of ΔH with Q.H.M. 89 at Brodsworth on 11 July 1950 using H variometer no. 12. Readings in the 0 (meridian) position were taken before and after the $+2\pi$ and -2π readings to check that the Q.H.M. 89 and the Q.H.M. 88, used as a declinometer, were following each other, sets with closing errors greater than 30 sec. being rejected. Each group of four rows gives one complete determination of H. The reduction of the variometer readings to provide the correction for the daily variation dH is given in columns 16 to 19 using a temperature coefficient (K) of $-0{\cdot}7\,\gamma/^\circ$ C and a sensitivity of $16{\cdot}3\,\gamma$/div. The value of H before and after correction for the daily variation is given in columns 15 and 21, column 22 contains the mean values of H at the different sites, and the two values of ΔH obtained from the mean differences of the fields at the surface and underground are given in column 23.

TABLE 13

colliery	latitude and longitude and National Grid co-ordinates of underground site	National Grid co-ordinates of surface site	depth of underground site (ft.)	height above O.D. of surface site (ft.)
Hickleton Main (nr. Doncaster, Yorks)	53° 32′ 42″ N 01° 17′ 36″ W 44684056	44704065	2520	210
Nook (nr. Leigh, Lancs)	53° 29′ 50″ N 02° 28′ 00″ W 36924004	36914009	2930	100
Astley Green (nr. Leigh, Lancs)	53° 28′ 40″ N 02° 26′ 48″ W 37053980	36914009	2859	64
Cadeby Main Denaby Main (nr. Doncaster, Yorks)	53° 29′ 57″ N 01° 13′ 44″ W 45094003	45134017	2600	300
Brodsworth Main	53° 33′ 50″ N 01° 11′ 47″ W 45294079	45274080	2485	135

The results of all the ΔZ measurements are given in table 14 and those of ΔH in table 15. Columns 2 and 3 in both tables give the positions of the surface and underground bases. Column 4 gives the variation with depth computed from the difference of the field at the surface at the beginning of the day and the field in the mine with a correction for the daily variation. Column 5 gives the same quantity computed from the field at the surface at the end of the day and the field in the mine. The mean of all the values of this quantity at each colliery is shown in column 6 together with its standard deviation. The determination of the temperature coefficients of the La Cour instruments gave standard deviations of the mean values of $0{\cdot}02\,\gamma/^\circ$ C for the B.M.Z. and $0{\cdot}004\,\gamma/^\circ$ C for the Q.H.M. Only the former is important over the temperature range encountered in the measurements, and an allowance is made for it in column 6 of table 14. Columns 7 to 12 give the various corrections, together with their standard errors, which it is necessary to apply to the quantity in column 6 to obtain the true value of the variation with depth. E_1 corrects the field at the underground base to the mean value obtained from the magnetic survey of the passage, excluding the effect of disturbances obviously due to buried iron. E_2 is the correction for the effect of iron in neighbouring roadways. E_3 is the correction for the effect of the permanent and induced magnetism of the surrounding medium. E_4 is the correction for the difference between the actual field at the surface base and the smoothed value at that point obtained from the surface survey results. This correction is large for the Brodsworth, Nook and Astley Green results, because it was found more convenient to choose a surface base situated so near the colliery

TABLE 14

1	2	3	4	5	6	7	8	9	10	11	12	13
						underground corrections			surface corrections			
date	surface base	underground base	ΔZ down	ΔZ up	mean ΔZ	E_1	E_2	E_3	E_4	E_5	E_6	corrected ΔZ
21. ix. 49	Hickleton II	Hickleton 300 ft.	+ 6	+12	+ 7 ± 1·7	+1 ±0·4	+ 3	0	− 3 ± 2·1	+5 ±0·2	−1	+12 ± 2·7
22. ix. 49			+ 6	+ 4								
6. ix. 49	Nook II	Nook 120 ft.	+35	+35	+32 ± 1·5	+1 ±0·7	0	0	−20 ± 2·1	0 ±0·2	0	+13 ± 2·7
14. ix. 49			+24	+25								
3. x. 49			+32	+39								
3. x. 49			+35	+34								
21. ii. 49			+34	+30								
17. iii. 49	Nook II	Astley Green 100 ft.	+38	+36	+34 ± 1·5	+1 ±0·5	+ 1	0	−20 ± 2·1	+1 ±0·9	0	+17 ± 2·8
20. iv. 49			+32	+37								
16. viii. 49			+31	+29								
4. iv. 50	Cadeby I	Cadeby 260 ft.	+ 1	+ 6	+ 4 ± 0·7	0 ±0·3	+ 1	0	0 ± 2·1	+8 ±0·3	−1	+12 ± 2·3
20. vi. 50			+ 5	+ 4								
5. vii. 50			+ 2	+ 6								
6. vii. 50			+ 4	+ 2								
4. vii. 50	Brodsworth I	Brodsworth 101 ft.	+45	+46	+46 ± 0·6	0 ±0·7	+17 ± 4	0	−57 ± 2·1	0 ±0·2	−1	+ 5 ± 4·6
4. vii. 50			+45	+46								
7. vii. 50			+48	+46								
7. vii. 50			+47	+48								

TABLE 15

1	2	3	4	5	6	7	8	9	10	11	12	13
						underground corrections			surface corrections			
date	surface base	underground base	ΔH down	ΔH up	mean ΔH	E_1	E_2	E_3	E_4	E_5	E_6	corrected ΔH
22. iv. 49	Hickleton II	Hickleton 300 ft.	+18	+13	+20 ± 1·0	+3 ±0·9	−1	0	0 ± 2·1	− 5 ± 0·2	0	+17 ± 2·5
25. iv. 49			+19	+22								
23. i. 50			+23	—								
24. i. 50			+20	+22								
25. i. 50			+22	+19								
4. viii. 49	Nook II	Nook 120 ft.	+ 4	+ 1	+ 3 ± 0·7	−2 ±0·6	0	0	+ 7 ± 2·1	− 2 ± 0·2	+2	+ 8 ± 2·3
8. viii. 49			+ 2	+ 3								
8. viii. 49			+ 2	+ 5								
18. vii. 49	Nook II	Astley Green 80 ft.	+11	+15	+12 ± 0·8	0 ±0·1	−1	0	+ 7 ± 2·1	−11 ± 0·9	+2	+ 9 ± 2·4
21. vii. 49			+10	—								
28. vii. 49			+14	+12								
15. ix. 49			+ 9	+11								
3. iv. 50	Cadeby I	Cadeby 260 ft.	+24	+24	+24 ± 0·5	0 ±0·5	−1	0	+ 4 ± 2·1	− 7 ± 0·3	0	+20 ± 2·2
5. iv. 50			+24	+26								
21. vi. 50			+24	+26								
23. vi. 50			+22	+22								
10. vii. 50	Brodsworth I	Brodsworth 101 ft.	−35	−35	−37 ± 1·3	0 ±0·5	−3 ±1·5	0	+61 ± 2·1	0 ± 0·2	0	+21 ± 2·9
11. vii. 50			−34	−34								
12. vii. 50			−34	−34								
13. vii. 50			−41	−40								
13. vii. 50			−40	−46								

that the field was disturbed and to correct the measurement from the data of the surface survey. The standard error of this quantity is taken to be the standard error of a single measurement in the survey determined from the histogram of closing errors. E_5 corrects the value of smoothed field at the surface base to the point vertically above the underground base. Its standard error is determined by multiplying the standard error in the field gradients given in table 9 by the difference in grid co-ordinates of the points. E_6 is the correction to remove the effects of the anomalous gradient (§ 2). Its standard error is found by multiplying the standard error in the field gradients given in table 9 by the depth of the mine. The final value of the variation with depth is given for each site in column 13, the standard error being the square root of the sum of the variances of the quantities in columns 6 to 12.

8. Discussion of results

Measurements were attempted at Nantawlaeth (South Wales), Llay Main and Gresford (North Wales), Bold (Lancashire) and Snowdown (Kent), but roadways sufficiently free of disturbance for significant measurements to be made could not be found. A suitable site was found at the Vane Tempest Colliery, County Durham, but the surface survey was very disturbed, changes of 30γ occurring in a kilometre so that no significant measurement was possible. Earlier measurements at the Parsonage Colliery, Leigh, Lancashire (briefly reported by Chapman 1948*b*), indicated a negative value of ΔH but twice that predicted by the 'distributed' theory. This underground site was too near the outskirts of the town of Leigh for a surface survey of adequate size to be made. Thus the corrections E_4, E_5 and E_6 cannot be estimated, and therefore the results cannot be regarded as valid.

Hales & Gough (1947) reported a negative value of ΔH in the Blyvooruitzicht Gold Mine in the Witwatersrand, South Africa. The region is magnetically disturbed, a magnetic traverse in Z about 2 km. from the site revealing a gradient of $60\,\gamma$/km. which is by no means linear (Krahmann 1936, traverse no. E 6–99 of plate XVII). It is thus clear from the considerations of this paper that the values of ΔH would be anomalous and difficult to correct.

In a later and more extensive investigation (described in a paper read at the Geophysical Conference at the Bernard Price Institute of Geophysical Research, July 1949), Hales & Gough showed that after making allowance for all the known magnetic bodies in the area, entirely different results for the radial variation were obtained in two different sections of the mine. The discrepancy could not be accounted for on the basis of experimental error, and must therefore be due to the presence of a magnetic body other than those for which allowance had been made. Hales & Gough therefore concluded that their work at Blyvooruitzicht gave no useful information with regard to the radial variation of the earth's field.

The positions of the five mines where a complete set of measurements were made and the depths of the cover are given in table 13. Table 16 summarizes the results obtained. The mean values of ΔZ and ΔH from the three mines in Yorkshire are $+11{\cdot}7$ and $+20{\cdot}3\gamma$ respectively, much closer to the values predicted by the core theory than to those predicted by the distributed theory. The values at each of the mines differ from these means by less than twice the standard error of the experimental measurements and can be regarded as not significantly different. The mean values of ΔZ and ΔH from the two mines in Lancashire, $+15{\cdot}0$ and $+8{\cdot}5\gamma$, are very close to the values predicted by the core theory. The significance of the difference between the mean values in Lancashire and Yorkshire can be tested by the t test (Fisher 1950, § 24·1). This test shows that the differences between the means of Z and of H for Lancashire and Yorkshire ($+4·3$ and $-11·8\gamma$ respectively) are significant at the 55 and 2 % level respectively.

TABLE 16. SUMMARY OF RESULTS

		theoretical			experimental	
colliery	depth (ft.)	ΔZ	ΔH (core)	ΔH (distributed)	ΔZ	ΔH
Hickleton	2500	+16	+7	−15	$+12\pm2·7$	$+17\pm2·5$
Cadeby	2600	+16	+7	−15	$+12\pm2·3$	$+20\pm2·2$
Brodsworth	2500	+16	+7	−15	$+5\pm4·6$	$+21\pm2·9$
Astley Green	2900	+18	+8	−17	$+17\pm2·8$	$+9\pm2·4$
Nook	2900	+18	+8	−17	$+13\pm2·7$	$+8\pm2·3$
average (referred to 2900 ft.)		+18	+8	−17	+13	+16

This regional difference might be thought to be due to the magnetic effects of the sedimentary strata, as the average anomaly, computed in § 3, was found to be of the order of 6γ. But the mines, both in Yorkshire and Lancashire, are spaced apart by about 5 miles, a distance very much greater than that taken in this calculation as the maximum extent of any one strata. Thus if the regional difference were due to this cause, a comparable spread in the values of ΔZ and ΔH within each group of mines would be expected but it does not appear in this case.

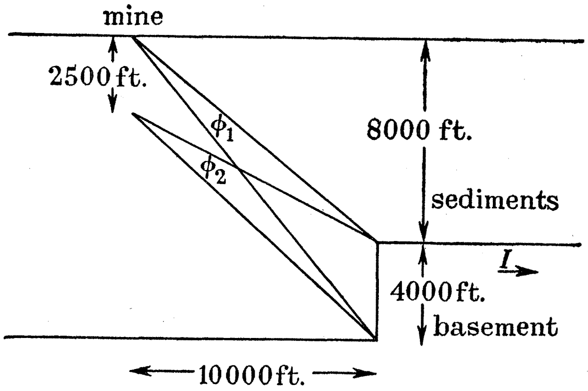

FIGURE 22. North-south section in south-west Yorkshire.

It is therefore concluded that the measurements in Yorkshire are affected by non-linear gradients arising from the igneous basement; the evidence discussed in § 3 indicating that its depth is not too great for such an effect to be just detectable. As the sediments in Yorkshire lie horizontally and are not greatly faulted, the structure of the basement may be expected to be revealed in the gravity surveys. These (White 1948) reveal a positive gravity ridge having a maximum of about 10 mgals, of 10 km. width, lying west-north-west to east-south-east, 10 km. to the north of Hickleton. Rucker & Thorpe (1890, 1896) showed the presence of a line source of magnetic disturbance in almost this position. Assuming a difference in density between the sediments and basement rocks of 0·2, the feature would be a ridge in the basement of about 4000 ft. high as shown in figure 22. Assuming the basement were polarized with horizontal intensity I in the normal direction, the H field in the Yorkshire mines should be augmented by an amount of the order of

$$2I(\phi_2-\phi_1)=+12\gamma \quad (I=10^{-3},\ \phi_1=11^\circ \text{ and } \phi_2=15^\circ),$$

which is sufficient to explain the anomaly. Seismic methods (White 1948) showed that this structure (the Lindholme anticline) has a relief of over 1000 ft. at the presumed upper surface of the Carboniferous. If the relief of the basement was only of this order then the value of I assumed would have to be increased. A detailed study of this feature will appear elsewhere.

The Lancashire results show no divergence from the predicted values, and this may indicate a considerable thickness of Lower Palaeozoic sediments, the depth of the basement being greater than was indicated in §3, or the absence of surface features on a shallow basement or the presence of Pre-Cambrian sediments of the Longmynd type.

It is thus concluded that there is no evidence to suggest that the main geomagnetic field does not increase within the earth's mantle as an inverse cube law. Thus unless it can be shown that the considerations are incorrect on which the prediction of the variation with depth is based, the experiments must be regarded as decisive evidence against a fundamental origin of the main geomagnetic field.

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TABLE 12

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
			Q.H.M.88 as declinometer			Q.H.M.89									variometer reading $H12$							
			micrometers				micrometers															
place	time (B.S.T.)	position	A	B	mean dD	temp. (°C)	A	B	mean $-dD$	ϕ	log sin ϕ	mean KT	log H	H	temp. (°C)	(divisions)	KT	dH	$\overline{dH}$	$H-\overline{dH}$	$\overline{H}$	ΔH
odsworth surface	0947	0	238° 34′ 59″	058° 24′ 58″	00″		160° 14′ 32″	340° 29′ 37″	00″													
	0949	$+2\pi$	238 34 52	58 24 50	− 07	16·3	113 52 26	294 07 40	46° 21′ 54	46° 20′ 08″	9·859376	0·002918	9·242218	17467	18·2	23·7	−1	+ 1	− 1	17468		
	1001	-2π	238 34 01	58 24 02	− 57	16·4	206 31 48	026 47 07	46 18 20						17·0	23·5	0	− 3				
	1007	0	238 34 12	58 24 11	− 47		160 13 28	340 28 50	−08													
																					17468	
	1007	0	238 34 12	58 24 11	00		160 13 28	340 28 50	00													
	1014	$+2\pi$	238 34 00	58 23 55	− 14	16·7	113 51 03	294 06 26	46 22 10	46 20 53	9·859461	0·003008	9·242223	17467	16·8	23·6	0	− 1	− 1	17468		
	1021	-2π	238 33 35	58 23 35	− 36	17·0	206 32 30	026 47 48	46 19 36						16·9	23·6	0	− 1				
	1028	0	238 33 35	58 23 35	− 36		160 12 56	340 28 22	+06													
																						−34
odsworth underground, 101 ft.	1242	0	—	62 16 48	00		173 12 53	353 28 42	00													
	1250	$+2\pi$	—	62 16 58	+ 10	30·3	126 20 42	306 36 14	46 52 28	46 49 12	9·862851	0·005417	9·241242	17428	17·0	23·2	0	− 7	− 6	17434		
	1256	-2π	—	62 16 28	− 20	30·4	219 58 37	040 14 08	46 45 55						17·0	23·4	0	− 5				
	1304	0	—	62 16 29	− 19		173 12 00	353 27 52	−32													
																					17434	
	1310	0	—	62 16 30	00		173 12 01	353 27 52	00													
	1314	$+2\pi$	—	62 15 57	− 33	30·4	126 21 32	306 37 03	46 50 11	46 47 57	9·862698	0·005426	9·241404	17434	17·0	23·9	0	+ 3	0	17434		
	1320	-2π	—	62 16 14	− 16	30·4	219 57 30	040 13 17	46 45 43						17·0	23·6	0	− 2				
	1329	0	—	62 16 30	00		173 12 03	353 27 40	−05													
																						−34
odsworth surface	1616	0	243 05 47	62 55 42	00		186 48 18	007 03 18	00													
	1620	$+2\pi$	243 05 20	62 55 17	− 26	19·6	140 24 17	320 39 41	46 23 23	46 21 09	9·859500	0·003587	9·242763	17489	19·5	24·4	−2	+14	+21	17468		
	1628	-2π	243 04 40	62 54 38	−1′ 05	20·6	233 06 02	053 21 15	46 18 55						20·0	25·3	−2	+28				
	1633	0	243 04 39	62 54 34	−1′ 08		186 47 18	007 02 30	+14													
																					17468	
	1633	0	243 04 39	62 54 34	00		186 47 18	007 22 30	00													
	1643	$+2\pi$	243 04 55	62 54 51	+ 16	21·4	140 25 42	321 00 56	46 21 15	46 18 55	9·859230	0·003837	9·243283	17510	20·2	25·7	−3	+35	+42	17468		
	1656	-2π	243 05 10	62 55 10	+ 33	21·6	233 03 48	053 39 04	46 15 59						20·8	26·4	−3	+48				
	1701	0	243 05 00	62 55 03	+ 25		186 48 00	007 03 22	+22													